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MATHÉMATIQUES

SUR UN CERTAIN DÉVELOPPEMENT DES FONCTIONS MÉTAHARMONIQUES

Par I. N. VEKUA

(Présenté par N. Mouskhelichvili, de l'Académie, le 22.IX.1944)

1. On sait (1) qu'une fonction $u(x_1, \ldots, x_p)$ $(p \geqslant 1)$, analytique au point $(x_1^0, \ldots, x_p^0)$ est développable en série

$$u(x_1, \ldots, x_p) = \sum_{k=0}^{\infty} u_k(x_1, \ldots, x_p)\, r^{2k} \quad \left(r^2 = \sum_{i=1}^{p} (x_i - x_i^0)^2\right) \tag{1}$$

où $u_k(x_1, \ldots, x_p)$ sont des fonctions harmoniques au voisinage du point $(x_1^0, \ldots, x_p^0)$ que l'on appelle coefficients harmoniques de la fonction u. La série (1) converge absolument et uniformément à l'intérieur d'une certaine sphère assez petite et ayant pour centre le point $(x_1^0, \ldots, x_p^0)$. Pour $u \equiv 0$ tous les $u_k \equiv 0$, ce qui démontre l'unicité du développement (1).

Dans la présente Note en nous appuyant sur nos résultats antérieurs (2–5) nous établissons quelques résultats plus précis concernant le domaine de la convergence de la série (1) pour les fonctions vérifiant les équations différentielles de la forme

$$\Delta^n u + a_1 \Delta^{n-1} u + \ldots + a_n u = 0 \quad (n \geqslant 1) \tag{2}$$

où $a_1, \ldots, a_n$ sont des constantes, et Δ est l'opérateur de Laplace:

$$\Delta = \sum_{i=1}^{p} \partial^2 / \partial x_i^2.$$

2. Considérons d'abord les fonctions vérifiant l'équation

$$\Delta u + \lambda^2 u = 0 \quad (\lambda = \text{const}) \tag{3}$$

De telles fonctions, si elles sont continues ainsi que leurs dérivées partielles des deux premiers ordres, seront nommées fonctions métaharmoniques de paramètre λ. Etudions d'abord les développements en série de la forme (1) des fonctions métaharmoniques dépendant de deux variables x, y.

Si $u(x, y)$ est une fonction métaharmonique dans un domaine fini simplement connexe T et qui prend, en général, des valeurs complexes, elle peut, comme je l'ai démontré dans (2), être présentée dans la forme

$$u(x, y) = \varphi(z) - \int_0^z \frac{\partial}{\partial t} J_0\left(\lambda \sqrt{z(z-t)}\right) \varphi(t)\, dt +$$

$$+ \overline{\psi(z)} - \int_0^z \frac{\partial}{\partial t} J_0\left(\lambda \sqrt{z(\bar{z}-\bar{t})}\right) \overline{\psi(t)}\, d\bar{t} \tag{4}$$

où $z = x + iy$, et φ et ψ sont des fonctions holomorphes dans le domaine T, qui sont liées à la fonction u de la manière suivante:

$$\varphi(z) = u\left(\frac{z}{2}, \frac{z}{2i}\right) - \frac{1}{2}u(0,0); \quad \psi(z) = \overline{u\left(\frac{\bar{z}}{2}, \frac{i\bar{z}}{2}\right)} - \frac{1}{2}\overline{u(0,0)} \tag{5}$$

L'origine des coordonnées ($z = 0$) appartient au domaine T, et la barre désigne la valeur complexe conjuguée.

En intégrant par parties on obtient de (4)

$$u(x, y) = \sum_{k=0}^{m} u_k(x, y)\, r^{2k} + R_m(x, y) \tag{6}$$

où

$$u_0(x, y) = \varphi(z) + \overline{\psi(z)} \tag{7}$$

$$u_k(x, y) = \frac{(-1)^k \left(\frac{\lambda}{2}\right)^{2k}}{k!\,(k-1)!} \left[\frac{1}{z^k}\int_0^z (z-t)^{k-1}\varphi(t)\,dt + \frac{1}{z^k}\int_0^{\bar{z}} (\bar{z}-\bar{t})^{k-1}\bar{\psi}(t)\,d\bar{t}\right]$$

$$(k = 1, 2, \ldots)$$

$$R_m(x, y) = (-1)^m \int_0^z \frac{\partial^{m+2}}{\partial t^{m+2}} J_0(\lambda\sqrt{z(z-t)})\,dt \int_0^t \frac{(t-\sigma)^m}{m!}\varphi(\sigma)\,d\sigma +$$

$$+ (-1)^m \int_0^{\bar{z}} \frac{\partial^{m+2}}{\partial t^{m+2}} J_0(\lambda\sqrt{z(\bar{z}-t)})\,d\bar{t} \int_0^{\bar{t}} \frac{(\bar{t}-\bar{\sigma})^m}{m!}\bar{\psi}(\sigma)\,d\bar{\sigma} \tag{8}$$

Démontrons maintenant que sur chaque ensemble fermé A, appartenant à T et n'ayant pas de points communs avec sa frontière, R_m tend uniformément vers zéro pour $m \to \infty$.

Enfermons l'ensemble A à l'intérieur d'un certain domaine A_0 polygonal fermé, situé dans le domaine T et n'ayant pas de points communs avec sa frontière. Soit d le diamètre du domaine T, et l le périmètre de A_0. Soit $z \in A_0$. Il est alors facile de voir que dans les intégrales qui se trouvent dans (7) et (8) on peut prendre pour chemins d'intégration des courbes partiellement unies dont les longueurs ne surpassent pas $d + l$. D'ailleurs, les fonctions φ at ψ sont uniformément bornées sur A_0. Nous aurons donc

$$\left|\int_0^z (t-\sigma)^k \varphi(\sigma)\,d\sigma\right| \leqslant Md^k, \quad \left|\int_0^{\bar{z}} (\bar{t}-\bar{\sigma})^k \overline{\psi(\sigma)}\,d\bar{\sigma}\right| \leqslant Md^k \tag{9}$$

où z et t sont des points quelconques de A_0 et M une constante positive. Ensuite en utilisant la formule

$$J_0(\omega) = \frac{1}{2\pi i}\int_{|\zeta|=1} \zeta^{-1} e^{\zeta - \frac{\omega^2}{4\zeta}}\,d\zeta \tag{10}$$

on obtient tout de suite l'évaluation

$$\left|\frac{\partial^k}{\partial t^k} J_0(\lambda\sqrt{\bar{z}(z-t)})\right| \leqslant Nd^k \tag{11}$$

où N est une constante positive qui ne dépend ni de z, ni de t ($z, t \in A_0$). En vertu de (9) et (11) on obtient de (8) immédiatement

$$|R_m(x, y)| \leqslant \frac{2MN(d+l)\,d^{2m+2}}{m!} \tag{12}$$

Il suit immédiatement de cette inégalité que R_m tend uniformément vers zéro sur A_0 pour $m \to \infty$, ce qui démontre notre affirmation. En vertu de la proposition démontrée, il résulte de (8)

$$u(x, y) = \sum_{k=0}^{\infty} u_k(x, y)\, r^{2k} \tag{13}$$

Ainsi nous avons établi le

Théorème 1. *Si $u(x, y)$ est une fonction métaharmonique dans un domaine fini simplement connexe T, elle peut être développée en une série* (13), *où les u_k sont des fonctions harmoniques que l'on détermine par les formules* (5) *et* (7). *Cette série converge absolument et uniformément à l'intérieur du domaine T, et on peut la dériver terme à terme autant de fois que l'on voudra.*

3. Si $u(x_1, \ldots, x_p)$ est une fonction métaharmonique dans un domaine étoilé D ayant pour centre l'origine des coordonnées, alors on sait (4) qu'il est possible de la représenter dans la forme

$$u(x_1, \ldots, x_p) = u_0(r, \theta_1, \ldots, \theta_{p-1}) - \int_0^r \left(\frac{\rho}{r}\right)^{\frac{p-2}{2}} \frac{\partial}{\partial \rho} J_0(\lambda \sqrt{r(r-\rho)}) \times$$

$$\times\, u_0(\rho, \theta_1, \ldots, \theta_{p-1})\, d\rho \tag{14}$$

où u_0 est une fonction harmonique dans D qui est déterminée d'une manière unique au moyen de u; $r, \theta_1, \ldots, \theta_{p-1}$ sont les coordonnées polaires du point $(x_1, \ldots, x_p)$. Dans la formule (14) l'intégrale est prise le long du rayon qui joint l'origine des coordonnées au point $(x_1, \ldots, x_p)$.

En intégrant par parties nous obtenons de (12)

$$u(x_1, \ldots, x_p) = \sum_{k=0}^{m} u_k(x_1, \ldots, x_p)\, r^{2k} + R_m(x_1, \ldots, x_p) \tag{15}$$

où

$$u_k(x_1, \ldots, x_p) = \frac{(-1)^k \left(\frac{\lambda}{2}\right)^{2k}}{k!\,(k-1)!} \frac{1}{r^{k+q}} \int_0^r (r-\rho)^{k-1} \rho^q\, u_0(\rho, \theta_1, \ldots, \theta_{p-1})\, d\rho \tag{16}$$

$$\left(k = 1, 2, \ldots, \quad q = \frac{p-2}{2}\right)$$

$$R_m(x_1, \ldots, x_p) =$$

$$= (-1)^m r^{-q} \int_0^r \frac{\partial^{m+1}}{\partial \rho^{m+1}} J_0\left(\lambda \sqrt{r(r-\rho)}\right) d\rho \int_0^{\rho} \frac{(\rho - \rho_1)^{m-1}}{(m-1)!} \rho_1^q\, u_0(\rho, \theta_1, \ldots, \theta_{p-1})\, d\rho \tag{17}$$

On obtient facilement de la dernière formule, que R_m tend uniformément vers zéro pour $m \to \infty$ sur chaque ensemble fermé appartenant à D et n'ayant pas de points communs avec sa frontière. C'est pourquoi on obtient de (15)

$$u(x_1, \ldots, x_p) = \sum_{k=0}^{\infty} u_k(x_1, \ldots, x_p)\, r^{2k} \tag{18}$$

où les u_k sont des fonctions harmoniques dans D déterminées par les formules (16). Ainsi nous avons établi le théorème suivant:

Théorème 2. *Si $u(x_1, \ldots, x_p)$ est une fonction métaharmonique dans un domaine étoilé D, on peut la développer en série* (18) *qui converge absolument et uniformément à l'intérieur de D, et on peut la dériver terme à terme autant de fois que l'on voudra.*

Il est évident que dans le cas où $p=2$ ce théorème est un cas particulier du théorème 1.

4. Ecrivons l'équation (2) dans la forme [5]

$$\Delta^k(\Delta+\varkappa_1)^{k_1}\ldots(\Delta+\varkappa_m)^{k_m}u=0 \tag{19}$$

Chaque solution de cette équation peut, comme on sait [5], être présentée dans la forme

$$u=v+\sum_{i=1}^{m}\sum_{s=0}^{k_i-1}r_s\frac{\partial^s u_{is}}{\partial r_s} \tag{20}$$

où v est une fonction k-harmonique c'est-à-dire une fonction vérifiant l'équation $\Delta^k v=0$, et les u_s sont des fonctions métaharmoniques de paramètre $\varkappa_i$.

Considérons d'abord le cas $p=2$. Soit $u(x, y)$ une solution régulière de l'équation (2) dans un domaine fini simplement connexe T. En vertu du théorème 1 les fonctions métaharmoniques u_{is} qui figurent dans (20) peuvent être développées en séries de la forme (13). Il est d'ailleurs facile à remarquer que les fonctions $r^s\frac{\partial^s u_{is}}{\partial r^s}$ peuvent aussi être présentées dans D par des séries analogues; à cet effet il suffit d'effectuer sur la série (13) l'opération $r^s(\partial^s/\partial r^s)$ et de tenir compte de ce que si w est une fonction harmonique, la fonction $r^k(\partial^k w/\partial r^k)$ est aussi harmonique. D'ailleurs, en tenant compte de ce que la fonction k-harmonique v est de la forme

$$v=v_0+v_1r^2+\ldots+v_{k-1}r^{2k-2}$$

où $v_0, v_1, \ldots, v_{k-1}$ sont des fonctions harmoniques, on obtient de (20) le théorème suivant.

Théorème 3. *Si la fonction $u(x, y)$ est une solution régulière de l'équation* (2) *dans un domaine fini simplement connexe T, elle peut être développée dans ce domaine en une série de la forme* (1) *absolument et uniformément convergente.*

En vertu du théorème 2 on obtient d'une manière tout à fait analogue de la formule (20) le

Théorème 4. *Si la fonction $u(x_1,\ldots,x_p)$ est une solution régulière de l'équation* (2) *dans un domaine étoilé D, elle peut être développée dans ce domaine en une série de la forme* (1) *absolument et uniformément convergente.*

Il est évident que dans le cas où $p=2$, le théorème 4 est un cas particulier du théorème 3.

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LITTÉRATURE CITÉE

[1] N. Cioranescu, Bull. Soc. math. de France, LXV, 41 (1937). [2] I. N. Vekua, C. R. Acad. Sci. USSR, XVI, No. 3 (1937). [3] И. Н. Векуа, Сообщ. АН ГССР, IV, 5 (1943). [4] И. Н. Векуа, Сообщ. АН ГССР, III, 4 (1942). [5] И. Н. Векуа, Тр. Тбилисск. мат. ин-та, XII, 105 (1943).

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MATHEMATICS

ON THE THEORY OF NON-ASSOCIATIVE n-GROUPS SATISFYING POSTULATE K

By S. A. TCHOUNIKHINE

(Communicated by O. J. Schmidt, Member of the Academy, 12.X.1944)

1. In this note we consider non-associative Dörnte's n-groups* satisfying the so-called «postulate K»**. A detailed exposition of the results obtained will be published elsewhere.

2. A set $\mathfrak{G} = \{a_m, a_r, \ldots\}$ will be called n-group satisfying postulate K, if the following conditions are fulfilled.

1) With every «word»-figure $a_{m_1} a_{m_2} \ldots a_{m_n}$, having all of its n «places» filled up by elements of $\mathfrak{G}$, an element a_m of $\mathfrak{G}$ is uniquely associated which is the «result» of the operation.

The operation being unique, no ambiguity can arise if the same symbol is used for the operation and for its result, so that we can write

$$a_m = a_{m_1} \; a_{m_2} \ldots \; a_{m_n}$$

2) Postulate K. To every place $i \leqslant n$ there corresponds a place j (coinciding with or different from i) such that the following relation is valid:

$$a_{m_1} \; a_{m_2} \ldots a_{m_{i-1}} \; (a_{m_i} a_{m_{i+n-1}}) \; a_{m_{i+n}} \ldots a_{m_{2n-1}} =$$
$$= a_{p_1} a_{p_2}, \ldots, a_{p_{j-1}} (a_{p_j} \ldots a_{p_{j+n-1}}) \; a_{p_{j+n}} \ldots \; a_{p_{2n-1}}$$

where the sequence of $2n-1$ symbols $a_{p_1}, a_{p_2}, \ldots, a_{p_{2n-1}}$ is obtained from the sequence $a_{m_1}, a_{m_2}, \ldots, a_{m_{2n-1}}$ by a substitution θ which depends only on i and j, but not on the elements of $\mathfrak{G}$ represented by the symbols. The places i and j are called associative with one another.

All the places in an operation formula are thereby divided into classes of mutually associative places.

3) The elements a_m and $a_{m_1}, \ldots, a_{m_{i-1}}, a_{m_{i+1}}, \ldots, a_{m_n}$ uniquely determine the element a_{m_i} for every i, i. e. the operation is uniquely reversible with respect to any place in the operation formula.

In what follows the symbols ω_N, $\omega^*_{N^*}$, ψ_P will denote the complexes (3) of the elements of $\mathfrak{G}$, the indices of which form the sets N, N^*, P, ... respectively. The symbol $(a)_M$ will mean that all the elements of the complex determined by the set of indices M are equal to a. If i is an index from M, then $(a)_i$ will indicate again that the element a of the complex has the index i in M.

This is a convenient notation whenever it is necessary to express that a given complex is the set-theoretical sum of some of its subcomplexes. The

* Dörnte (1) considered associative groups only.

** The term «postulate K» is due to A. K. Sushkevich, who has introduced a particular form of this postulate.

«multiplicative» form of representation of complexes as «words» does not afford this advantage.

Let us express postulate K in the symbols just introduced. Since everyone of the symbols $a_{m_1}, a_{m_2}, \ldots, a_{m_{2n-1}}$ in the left-hand side of the equality expressing postulate K, enters the right-hand side of the equality, all these symbols may be divided into the following 4 classes:

I. The symbols that are inside the parentheses both in the left-hand side and in the right-hand side of the equality.

Such symbols form a subcomplex ω_N of the complex formed by the elements inside the parentheses in the left-hand side, and a subcomplex $\omega^*_{N^*}$ of the analogous complex in the right-hand side.

II. The symbols that are outside the parentheses in the left-hand side of the equality, but stand inside the parentheses in the right-hand side of the equality.

Such symbols form a subcomplex ψ_P of the whole complex in the left-hand side of the equality and a subcomplex $\psi^*_{P^*}$ of the complex parenthesized in the right-hand side of the equality.

III. The symbols that are inside the parentheses in the left-hand side of the equality, but appear outside the parentheses in the right-hand side of the equality.

Such symbols form subcomplexes χ_R and $\chi^*_{R^*}$ with respect to the left-hand and the right-hand sides of the equality.

IV. The symbols that are outside the parentheses both in the left-hand side and in the right-hand side of the equality.

Such symbols form subcomplexes φ_S and $\varphi^*_{S^*}$ with respect to the left-hand side and right-hand side of the equality.

It is evident that N, N^*, P, P^*, R, R^*, S, S^* are subsets of the set $M=\{1, 2, \ldots, n\}$. Bearing in mind their definitions, we denote these sets by

$$N=(i\theta j),\ N^*=(j\theta^{-1}i),\ P=[i\theta j]$$

Hence

$$P^*=M-(j\theta^{-1}i);\ R=M-(i\theta j);\ R^*=[j\theta^{-1}i];\ S=M-[i\theta j]-\{i\};$$
$$S^*=M-[j\theta^{-1}i]-\{j\}$$

The symbols of the same category determine, evidently, one-to-one mappings of everyone of the sets N, P, R, S onto N^*, P^*, R^*, S^*, respectively. Such mappings, as well as the elements corresponding to one another and the complexes (in pairs) ω_N, $\omega^*_{N^*}$, ψ_P, $\psi^*_{P^*}$, χ_R, $\chi^*_{R^*}$, φ_S, $\varphi^*_{S^*}$ will be called reciprocal.

As has been agreed upon previously, complexes of elements of $\mathfrak{G}$ with the set of indices M are considered to indicate the results of the operation in $\mathfrak{G}$. Therefore equalities between such complexes are not to be understood in the set-theoretical sense ([3]), but rather as identities of the results of operations represented by these complexes. Hence, summing up the symbol of all the four categories, we may write postulate K as the following conditional equality between the complexes:

$$(\omega_{(ij)}+\chi_{M-(i\theta j)})_i+\psi_{[i\theta j]}+\varphi_{M-[ij]-\{i\}}=(\omega^*_{(j\theta^{-1}i)}+\psi^*_{M-(j\theta^{-1}i)})_j+$$
$$+\chi^*_{[j\theta^{-1}i]}+\varphi^*_{M-[j\theta^{-1}i]-\{j\}} \qquad (1)$$

3. The following theorems are examples of the results that can be obtained by means of the formula (1). Theorem 1 is analogous to the theorem on right and left units in a group. Theorem 2 is analogous to the theorem on the decomposition of a group with respect to a modulus.

Theorem 1. *If the places i and j are associative, then the equality* $(a)_{M-(i\vartheta j)} + \varepsilon_{(i\vartheta j)} = a$, *where a is an arbitrary element of* $\mathfrak{G}$ *and* $\varepsilon_{(i\vartheta j)}$ *is a complex, all the elements of which enter* $\mathfrak{G}$, *implies* $(a)_{M-(j^{\vartheta^{-1}}i)} + \varepsilon^*_{(j^{\vartheta^{-1}}i)} = a$. *In other words, if* $\varepsilon_{(i\vartheta j)}$ *is a «unit» complex with respect to a, then the reciprocal complex* $\varepsilon^*_{(j^{\vartheta^{-1}}i)}$ *has the same property.*

Suppose $\mathfrak{H} \subset \mathfrak{G}$ and let $\mathfrak{H}$ satisfy the same postulates with respect to the operation in $\mathfrak{G}$ as are satisfied by $\mathfrak{G}$ itself. Then $\mathfrak{H}$ is called subgroup of $\mathfrak{G}$.

By the symbol $(\mathfrak{H})_N$ we shall understand the totality of all complexes of the form ε_N, the elements of which belong to $\mathfrak{H}$.

Theorem 2. *If the places i and j in an operation formula are associative, and the sets of elements of* $\mathfrak{G}$ *represented by the formulae* $(\mathfrak{H})_{(i\vartheta j)} + \eta_{M-(i\vartheta j)}$ *and* $(\mathfrak{H})_{(i\vartheta j)} + \zeta_{M-(i\vartheta j)}$ *have an element in common, then the sets of elements* $\mathfrak{G}$ *represented by the formulae* $(\mathfrak{H})_{M-[j^{\vartheta^{-1}}i]} + \eta^*_{[j^{\vartheta^{-1}}i]}$ *and* $(\mathfrak{H})_{M-[j^{\vartheta^{-1}}i]} + \zeta^*_{(j^{\vartheta^{-1}}i)}$ *coincide. Here* $\eta_{M-(i\vartheta j)}$ *and* $\zeta_{M-(i\vartheta j)}$ *are any complexes of elements of* $\mathfrak{G}$ *with the set of indices* $M-(i\vartheta j)$.

From Theorem 2 follow some «decomposition theorems (with respect to a modulus)» which for lack of space will not be dwelled upon here.

Let us note the following particular cases of Theorem 2.

Theorem 3. *If i and j are associative and every element from* $M-(i\vartheta j)$ *coincides with its reciprocal from* $[j\vartheta i]$, *then the sets of elements of* $\mathfrak{G}$ *represented by the formulae* $(\mathfrak{H})_{(i\vartheta j)} + \eta_{M-(i\vartheta j)}$ *and* $(\mathfrak{H})_{(i\vartheta j)} + \zeta_{M-(i\vartheta j)}$ *either coincide or have no common element.*

It is easy to verify that the conditions of Theorem 3 are fulfilled for Dörnte's n-groups.

If the set of elements of $\mathfrak{G}$ given by the formula $(\mathfrak{H})_{(i\vartheta j)} + \eta_{M-(i\vartheta j)}$, where $\eta_{M-(i\vartheta j)}$ is an arbitrary complex of elements of $\mathfrak{G}$ with the set of indices $M-(i\vartheta j)$, coincides with the set of elements of $\mathfrak{G}$ given by the formula $(\mathfrak{H})_{M-[j\vartheta^{-1}i]} + \eta^*_{[j\vartheta^{-1}i]}$, then $\mathfrak{H}$ is said to be $[j^{-1}i]$ invariant in $\mathfrak{G}$.

Theorem 4. *If i and j are associative and* $\mathfrak{H}[j\vartheta^{-1}i]$ *is invariant in* $\mathfrak{G}$, *then the sets of elements of* $\mathfrak{G}$ *given by the formulae* $(\mathfrak{H})_{(i\vartheta j)} + \eta_{M-(i\vartheta j)}$ *and* $(\mathfrak{H})_{(i\vartheta j)} + \zeta_{M-(i\vartheta j)}$ *either coincide or have no common element.*

Theorem 5 (generalized Dörnte's theorem [2], Theorem 4). *For any a, b in* $\mathfrak{G}$ *we have* $(a)_{M-(i\vartheta j)-\{t\}} + (b)_t + \eta_{(i\vartheta j)} = b$, *provided* $(a)_{-(j\vartheta^{-1}i)} + \eta^*_{(j\vartheta^{-1}i)} = a$, *$t$ belongs to* $M-(i\vartheta j)$ *and i is reciprocal to t in* $[j\vartheta^{-1}i]$.

4. Complexes with ordered sets of indices and «words» are useful in the theory of associative groups, because they facilitate the realisation and study of various transformations by which one iteration of the group operation is carried into another. As for the construction of the theory of n-groups satisfying postulate K, the use of ordered complexes and words is no longer essential and may be avoided.

In fact, let us introduce the operation of enlargement of complexes, which consists in considering a subcomplex of a complex as a new element with a definite index. The inverse process will be called «refinement» of a complex. An operation consisting of a finite number of «enlargements» and «refinements» will be called elementary.

Consider now a set $\mathfrak{G}$, whose elements will be called «objects of the operation», and a «fundamental» set M, not necessarily ordered, consisting of n elements. With every complex η_M, the elements of which are objects of the operation, an object from $\mathfrak{G}$ is associated by means of a definite rule based upon the nature of elements and the indices of the complex. This new element of $\mathfrak{G}$ will be called «result» of the operation applied to the elements of $\mathfrak{G}$ entering η_M, or, in other words, «value» of the complex η_M. Let us agree, in case we have a one-to-one mapping g of a set N on to M, to assign to any complex ζ_N consisting of elements of $\mathfrak{G}$ the value of the complex obtained from ζ_N by replacing its indices by the corresponding (under the mapping g) elements of M. Complexes which have got values in this way are called admissible. We will also agree to identify every admissible complex with the result of the operation represented by this complex. Under this condition an elementary transformation will be called admissible elementary transformation.

if it uses admissible complexes only. Suppose a set R of complexes is given, everyone of which can be carried, by a definite admissible elementary transformation, into an admissible complex with n terms. The value of the latter is supposed to belong to R.

Now assume that this operation is such that no complex in R has its value changed by any admissible elementary transformation. This is expressed by equalities of the form (1). Such equalities also indicate which of the elements of a complex may be associated in admissible subcomplexes in order that all of its admissible elementary transformations be realizable.

Thus, if the order and the «words» are not used in the notation of complexes, postulate K acquires the sense of the associative law in a general form. Partial order (without transitivity) and polygonal figures similar to structure formulae in chemistry appear here to play a rôle analogous to that which order and «words» play in the theory of associative groups. The above constructions may also be applied to infinite complexes.

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REFERENCES

[1] W. Dörnte, Math. ZS., **29**, 1 (1928). [2] А. К. Сушкевич, Теория обобщенных групп, Харьков—Киев, 1937. [3] F. Hausdorff, Mengenlehre, 1937.

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MATHÉMATIQUES

REMARQUES SUR L'ÉTUDE DE L'APPROXIMATION PAR LES FONCTIONS RATIONNELLES DANS UN DOMAINE COMPLEXE. II

Par A. SCHAGINJAN

(Présenté par S. L. Soboleff, de l'Académie, le 26.I.1944)

1. Il est connu ([1],[2]) que la possibilité d'approcher en moyenne par polynômes dans un domaine dont le complémentaire n'est pas connexe dépend des propriétés métriques du domaine.

Récemment, l'ordre limite de la décroissance du poids a été indiqué ([3]) dans le cas où dans un domaine arbitraire, dont le complémentaire par rapport au plan n'est pas connexe, le système des polynômes correspondant à ce poids est complet au sens de l'approximation en moyenne.

Il est intéressant d'étudier le problème dans quelles classes concrètes de domaines le système des polynômes sans aucun poids est complet. Dans la présente Note nous résolvons le problème de l'approximation uniforme avec un poids pour polynômes considérés dans certains domaines dont les complémentaires ne sont pas connexes. Ensuite nous étudions pour ces domaines les problèmes voisines de l'approximation en moyenne, de l'approximation uniforme des fonctions continues arbitraires sur des continus qui ne divisent pas le plan, ainsi que le problème sur l'approximation simultanée dans plusieurs domaines.

2. Désignons par B le domaine situé dans le plan de la variable z et borné par deux circonférences C_1 et C_2 à tangente commune; on suppose que C_2 est à l'intérieur de C_1 et que le centre de C_1 est à l'origine des coordonnées.

Le point de contact de ces circonférences sera désigné par A.

On nommera domaine du type Δ_B un domaine arbitraire situé dans B et tel que la transformation $w = \sqrt{z}$ le transforme en un domaine à complémentaire connexe, où l'approximation uniforme par polynômes dans le domaine fermé est possible.

Désignons par ρ la distance d'un point arbitraire z au point double A.

On a alors les théorèmes suivants:

Théorème 1. *Si $f(z)$ est une fonction quelconque analytique dans un domaine du type Δ_B continue jusqu'au contour, sauf peut être au point double A, et vérifiant la condition*

$$\lim_{\rho \to 0} f(z)\, e^{-\frac{1}{\rho}} = 0$$

l existe une suite de polynômes $\{P_n(z)\}$ pour lesquels on a dans le domaine Δ_B

$$\sup_{n \to \infty} e^{-1/\rho^{-1\varepsilon}} |f(z) - P_n(z)| \to 0, \quad \varepsilon > 0 \text{ arbitraire.}$$

On peut en même temps indiquer une fonction, comme par exemple $\sqrt{z}$, *pour laquelle l'approximation*

$$\sup e^{-1/\rho^{1-\varepsilon}} |f(z) - P_n(z)| \to 0, \quad z \subset \Delta_B$$

est impossible.

Une affirmation plus générale que cette dernière est encore vraie.

Si pour une fonction $f(z)$ *régulière dans* Δ_B *on a*

$$\sup_{n\to\infty} e^{-1/\rho^{1-\varepsilon}} |f(z) - P_n(z)| \to 0$$

alors $f(z)$ *est analytiquement prolongeable à l'intérieur de la circonférence* C_2.

3. Désignons par B_2 la classe des fonctions régulières dans Δ_B et vérifiant la condition

$$\iint_{\Delta_B} |f(z)|^2 \, dx\, dy < \infty$$

Désignons par $T(\rho)$ la mesure linéaire des arcs que le domaine donné Δ_B découpe sur la circonférence de rayon ρ dont le centre est au point A.

Théorème 2. *Si* $T(\rho) < e^{-1/\rho^{1+\varepsilon}}$, $\varepsilon > 0$, *alors pour chaque fonction* $f(z)$ *de classe* B_2 *on a dans* Δ_B

$$\inf_{n\to\infty} \iint_{\Delta_B} |f(z) - P_n(z)|^2 \, dx\, dy = 0$$

et si

$$T(\rho) > e^{-1/\rho^{1-\varepsilon}}, \quad \varepsilon > 0$$

il existe un domaine Δ_B *et une fonction, par exemple* $\sqrt{z}$, *pour laquelle*

$$\inf_{n\to\infty} \iint_{\Delta_B} |\sqrt{z} - P_n(z)|^2 \, dx\, dy \neq 0$$

L'affirmation suivante a lieu.

Si dans Δ_B *on a pour chaque fonction de classe* $f(z)$ *l'approximation*

$$\inf_{n\to\infty} \iint_{\Delta_B} |f(z) - P_n(z)|^2 \, dx\, dy = 0$$

pourvu que $T(\rho) > e^{-1/\rho^{1-\varepsilon}}$, *alors* $f(z)$ *est régulière partout à l'intérieur de* C_1.

Le théorème 2 résulte du théorème 1.

4. Pour chaque fonction continue sur un ensemble borné partout non dense et ne divisant pas le plan l'approximation uniforme par polynômes est possible (4).

Le théorème 1 permet de généraliser le résultat de M. Lavrentieff en l'étendant à une certaine classe d'ensembles qui divisent le plan.

Théorème 3. *Sur chaque ensemble* B^* *partout non dense, situé dans* B *et qui est transformé par la fonction* $w = \sqrt{z}$ *en un ensemble qui ne divise pas le plan, pour chaque fonction continue* $f(z)$ *l'approximation*

$$\sup_{n\to\infty} e^{-1/\rho^{1+\varepsilon}} |f(z) - P_n(z)| \to 0, \quad \varepsilon > 0$$

est possible. Et il existe un ensemble du type du considéré et une certaine fonction continue sur cet ensemble, par exemple, $\sqrt{z}$ *sur* C_2, *pour laquelle l'approximation*

$$\sup_{n\to\infty} e^{-1/\rho^{1-\varepsilon}} |\sqrt{z} - P_n(z)| \to 0 \quad \varepsilon > 0$$

est impossible.

5. Dans le cas de deux domaines n'ayant pas de points communs l'approximation simultanée par polynômes est toujours possible dans le cas de l'approximation uniforme et de même dans le cas de l'approximation en moyenne.

Cette propriété reste encore vraie dans le cas des domaines ayant un point frontière commun (5).

Mais si les domaines ont au moins deux points frontières communs, alors les propriétés métriques des domaines jouent un rôle dans le problème que nous étudions.

On déduit du théorème 1 un certain critère métrique concernant le cas où deux domaines ont au moins deux points frontières communs.

Théorème 4. *Soient Δ'_B et Δ''_B deux domaines simplement connexes dans Δ_B et qui pour $w=\sqrt{z}$ se transforment en deux domaines ayant au plus un point frontière commun et admettant une approximation uniforme par polynômes dans chaque domaine fermé; alors quelles que soient les fonctions $f_1(z)$ et $f_2(z)$ régulières respectivement dans Δ'_B et Δ''_B, continues jusqu'au contour, sauf peut être, au point multiple A, et vérifiant la condition*

$$\lim_{\rho\to\infty} f_i(z)\,e^{-1/\rho}=0 \qquad (i=1,2)$$

l'approximation

$$\left\{ \sup_{\substack{n\to\infty \\ z'\subset\Delta_{B'}}} e^{-1/\rho^{1+\varepsilon}}\,|f_1(z')-P_n(z')| + \sup_{\substack{n\to\infty \\ z''\subset\Delta_{B''}}} e^{-1/\rho^{1+\varepsilon}}\,|f_2(z'')-P_n(z'')| \right\} \to 0$$

est possible, et les polynômes $P_n(z)$ dans les deux membres de la somme sont les mêmes.

Théorème 5. *Soient $f_1(z)$ et $f_2(z)$ deux fonctions de classe B_2 respectivement dans $\Delta_{B'}$ et $\Delta_{B''}$; alors si l'inégalité*

$$T(\rho) < e^{-1/\rho^{1+\varepsilon}}, \qquad \varepsilon>0$$

a lieu, on a l'approximation

$$\inf_{n\to\infty}\left\{ \iint_{\Delta_{B'}} |f_1(z)-P_n(z)|^2\,dx\,dy + \iint_{\Delta_{B''}} |f_2(z)-P_n(z)|^2\,dx\,dy \right\} \to 0$$

Et si l'on a

$$T(\rho) > e^{-1/\rho^{1-\varepsilon}}, \qquad \varepsilon>0$$

une telle approximation est, en général, impossible.

6. Désignons par D un domaine fini simplement connexe situé dans un plan découpé le long de la partie négative de l'axe réel, et qui par $w=\sqrt{z}$ est transformé en un domaine dont le complémentaire est connexe et où l'approximation uniforme dans un domaine fermé est possible. Désignons par δ la distance du point z à la partie négative de l'axe réel $-\infty < x \leqslant 0$.

Théorème 6. *Si $f(z)$ est régulière à l'intérieur de D, continue jusqu'au contour, sauf peut être aux points situés sur $-\infty < x \leqslant 0$, et vérifiant la condition*

$$\lim_{\delta\to\infty} f(z)\,e^{-e^{1/\delta}}=0$$

alors l'approximation

$$\sup_{n\to\infty} e^{-e^{1/\delta^{1+\varepsilon}}}\,|f(z)-P_n(z)| \to 0, \qquad \varepsilon>0$$

arbitraire est possible et il existe des fonctions, par exemple $\sqrt{z}$, pour lesquelles une telle approximation est impossible pour le poids $e^{-e^{1/\delta^{1-\varepsilon}}}$.

D'ailleurs, si pour une fonction quelconque de la classe indiquée l'approximation avec le poids $e^{-e^{1/\rho^{\delta-\varepsilon}}}$ *est possible, il en résulte déjà que cette fonction est régulière sur* $-\infty < x \leqslant 0$.

On en déduit des théorèmes analogues à 2—5; énonçons l'un d'eux.

Menons des droites parallèles à la partie négative de l'axe ox à distance δ de cet axe, et joignons par une circonférence de rayon δ leur extrémités près du point $z=0$. Désignons par $T(\delta)$ la mesure linéaire des segments que le domaine D découpe sur le contour ainsi formé.

Théorème 7. *Si* $T(\delta) < e^{-e^{1/\delta^{1+\varepsilon}}}$, $\varepsilon > 0$, *alors pour chaque fonction* $f(z)$ *de classe* B_2 *l'approximation*

$$\inf_{n\to\infty} \iint_D |f(z) - P_n(z)|^2 \, dx\, dy = 0$$

est possible dans D, *et si*

$$T(\delta) > e^{\,e^{1/\delta^{1-\varepsilon}}}$$

il existe dans le domaine D *des fonctions, par exemple* $\frac{1}{z}$, *pour lesquelles l'approximation*

$$\inf_{n\to\infty} \iint_D |f(z) - P_n(z)|^2 \, dx\, dy = 0$$

est impossible, si D *est un domaine convenablement choisi. Par exemple, on obtient un tel domaine en prenant des courbes issues du point* x_0 $(-\infty < x_0 < 0)$ *et ayant les équations*

$$\frac{\sin n\varphi}{r^n} = \pm\frac{\pi}{4}, \quad \frac{\sin n\varphi}{r^n} = \pm\frac{\pi}{8} \Bigg\} \quad \varphi = \arg(z - x_0), \quad r = |z - x_0|$$

où $n/(n+1) > 1-\varepsilon$, *et en joignant leurs extrémités par certains arcs qui contournent l'origine des coordonnées.*

La méthode de la démonstration des théorèmes 1 et 6 est donnée dans (6).

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LITTÉRATURE CITÉE

[1] M. Keldych, Rec. Math., **5**, No. 2 (1939). [2] A. Schaginjan, C. R. Acad. Sci. URSS, XXVII, No. 4 (1940). [3] M. Keldych, ibid., XXX, No. 9. (1941). [4] М. А. Лаврентьев, Труды Мат. ин-та им. В. А. Стеклова, V (1934). [5] A. Schaginjan, Bull. Acad. Sci. URSS, sér. mathém., **5**, No. 4—5 (1941) [6]. A. Schaginjan, C. R. Acad. Sci. URSS, XLV, No. 2 (1944).

Correction. Dans la Note précédente du même auteur, C. R, Acad. Sci, URSS, vol. XBIV, No. 2

			au lieu de	lire
page 48	ligne 2	d'en bas	sup $e^{-\lvert z\rvert^{1/2-\varepsilon}}$	sup $e^{-\lvert z\rvert^{1-\varepsilon}}$
» 49	» 6	d'en haut	sup $e^{-\lvert z\rvert^{1+\varepsilon}}$	sup $e^{-\lvert z\rvert^{1/2+\varepsilon}}$
» 49	» 8	» »	sup $e^{-\lvert z\rvert^{1-\varepsilon}}$	sup $e^{-\lvert z\rvert^{1/2-\varepsilon}}$
» 50	» 10	» »	sup $e^{-\frac{1}{\lvert z\rvert^{1-\varepsilon}}}$	sup $e^{-\frac{1}{\lvert z\rvert^{1/2-\varepsilon}}}$

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PHYSICS

ELEMENTARY ELECTROCRYSTALLIZATION PROCESSES

By K. M. GORBUNOVA and P. D. DANKOV

(Communicated by S. I. Vavilov, Member of the Academy, 2.IX.1944)

Elementary electrocrystallization processes (1) will be discussed here by taking as an example the isolated growth of a face of a single crystal, forming part of the electrode of an electrolytic cell. In order to eliminate variations in the current strength a very high resistance is inserted in the circuit in series with the cell. At the start of the process there is a rapid accumulation of excess ions at the cathode surface—in the double electric layer. If the over-voltage is high enough ($\eta > \eta_2$), the metallic ions begin to lose their charges and form a two-dimensional nucleus at the edge of the face, or at the respective trihedral angle.

This gives rise to an electrical current whose magnitude depends upon the speed with which the two-dimensional nuclei are formed (2):

$$I_2 = k_2' e^{-k_2''/\eta} \qquad (1)$$

where I_2, the intensity of the current, is proportional to the number of metallic ions deposited on the cathode per unit time; k_2' is a coefficient, and k_2'' another coefficient which expresses the relationship between the work of formation of the nuclei (A_2) and the over-voltage (η).

As the face grows, the two-dimensional nucleus forms practically at only one of the angles or edges. In order to simplify further reasoning let use assume that the same takes place in our case, too. This we can do without restraining the generality of our conclusions.

The current I_2 which arises as a result of the over-voltage is too small to consume all of the ions which enter the double electric layer; therefore, even at the moment when the first two-dimensional nucleus is forming, the over-voltage on the cathode continues to rise, although after the appearance of the two-dimensional nucleus ion consumption is accelerated and proceeds by deposition of chains of structural particles, which arrange themselves along the edges of the nucleus. The development of this new stage in the process affects the current in the circuit, inasmuch as a new current I_1 makes its appearance, this latter representing the ion consumption due to the formation of a chain along the edge of the two-dimensional nucleus

$$I_1 = k_1' e^{k_1''\eta} \qquad (2)$$

where k_1' is a coefficient varying with the height and breadth of the front of growth, and k_1'' is a constant coefficient for any given conditions of electrocrystallization.

However, in spite of the fact that the discharge current is connected with a less difficult process, *viz.* the deposition of chains of structural particles, its magnitude is still so low that the over-voltage not only does not decrease, but continues to rise owing to the further accumulation of positive ions in the double layer; and this, in its turn, increases the probability of formation of a two-dimensional nucleus and accelerates chain formation. Thereupon two p a r a l l e l processes begin to develop with avalanche-like acceleration: 1) formation of a new two-dimensional nucleus (on the one already formed), and 2) growth of chains along the edges of the two-dimensional crystals.

This formation and development of new two-dimensional nuclei results in the «piling up» of two-dimensional crystals*, which results in multi-stage layer formation, a phenomenon which has been noted by several different investigators and which manifests itself in visible movement of the front of the layer. The quantity of metal deposited per unit time in the process as a whole equals the sum of the amounts deposited in the two parallel processes mentioned and corresponds to the magnitude of the discharge current:

$$I_{\text{disch}} = I_2 + I_1 = k_2' e^{-k_2''/\eta} + k_1' e^{k_1''\eta} \tag{3}$$

At a certain moment of time t_1 (see figure), when the over-voltage on the cathode reaches its maximum, the ion discharge current becomes equal to the current in the external circuit, *i. e.* $I_{\text{disch}} = I_0$. This corresponds to a considerably increased metal ion consumption in crystallization. The height of the front of growth and the current continue to rise until the over-voltage on the cathode falls below some average value η_2 at which the time the two-dimensional nuclei take to form becomes comparable with the time a metal layer takes to spread over the face. The metal discharge current increases ($I_{\text{disch}} > I_0$), and reaching a maximum value at the instant when the two-dimensional nuclei stop forming, begins to fall together with the over-voltage and then approaches a constant value

$$I_0 = k_{01}' e^{k_1''\eta_0} \tag{4}$$

at which the layer grows stationarily over the crystal face. From this moment on the over-voltage η_0 and the coefficient k_{01} remain constant until the front of growth reaches the edge of the cystal.

In the figure the ligth line *Oabc* depicts the probable course of the over-voltage on the cathode through the stage of metal ion discharge described above. A repetition of this process will evidently go along *ca'b'c'*, *etc.*

To make it agree with observation, several essential corrections must be introduced into this scheme:

1) During such a periodical electrocrystallization the over-voltage on the cathode remains approximately constant.

2) Several different investigators have shown that the height of the front of growth and the velocity of its movement vary with the concentration of metal ions in solution and with the current in the circuit of the cell.

3) One of the authors of this paper ([3]) established that shifting the growing face in relation to the electrolyte changes the velocity and direction of movement of the front of growth.

This last fact showed quite clearly that there exists a close relationship between the elementary processes of growth front formation and its subsequent movement along the face, on the one hand, and the factors governing the supply of metal ions to this front, on the other. The possibility of changing the conditions of feeding of the growth front (with metal ions) by shifting the solution layer nearest to the electrode tangentially, finds an explanation in Prandtl's conclusions ([4]). Contrary to Nernst's opinion of the impossibility of shifting tangentially a thin bordering liquid layer, Prandtl found such a shifting to be possible. In view of these considerations, the above mentioned fact of the influence of movement on elementary layer formation on the face indicates that along the face there exists a metal ion concentration gradient, which decreases the effective over-voltage (η_Φ) through the mechanism of concentrational polarization (ψ).

In view of this a correction must be introduced into the current intensity equation for the formation of the front of growth

$$\eta\Phi = \eta - \psi\Phi \tag{5}$$

Equation (3) may then be rewritten as

$$I_{\text{disch}} = k_2' e^{-k_2''/(\eta - \psi\Phi)} + k_1' e^{k_1''(\eta - \psi_\Phi)} \tag{6}$$

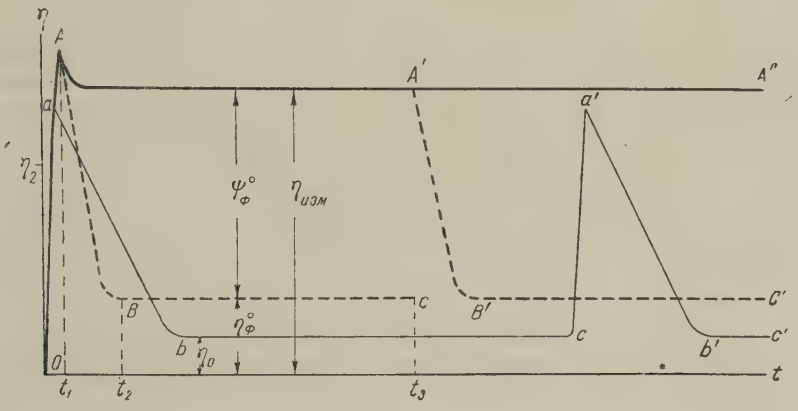
---

* The greater probability of the new two-dimensional nuclei being deposited upon those formed previously is due to the greater purity of the newly formed surfaces.

and expresses a more pronounced decrease in the effective over-voltage $(\eta - \psi_\Phi)$, while the over-voltage on the cathode as a whole (η) remains practically constant, if the dimensions of the face are small enough in comparison with the surface of the cathode. Obsiously, equation (4) may also be rewritten as:

$$I_0 = k'_{01} e^{k'_1 (\eta^0 - \psi^0_\Phi)} \qquad (7)$$

In this equation both η and ψ_Φ are constants: $\eta = \eta^0$ and $\psi_\Phi = \psi^0_\Phi$, with which the process is stationary until the front of growth reaches the edge of the face. At this moment the initial metal ion concentration at the initial edge and at other parts



Variation in cathode over-voltage and effective over-voltage at the front of growth with periodic electrocrystallization at the crystal face. Continuous heavy line, over-voltage measured at the cathode. Dotted line, effective over-voltage at the front of growth. Continuous light line, over-voltage at the cathode without concentrational polarization

of the face, which the front has already left, is restored by diffusion, and the effective over-voltage becomes $\eta_\Phi = \eta$. This makes the process of growth front formation repeat itself. The variations in the effective over-voltage at the front of growth with concentrational polarization taken into account is shown in the figure by the dotted lines $OABC$ (for the first period) and $A'B'C'$ (for the second and subsequent periods). The over-voltage at the cathode η is depicted by the line $OAA'A''$.

From analysing the schemes of electrocrystallization processes mentioned above we are permitted to draw the conclusion that they correctly describe the relation between face growth, height (layer thickness) changes and growth front spreading velocities, on the one hand, and current and electrolyte concentration, on the other, as observed experimentally.

The same scheme may be used to describe the growth process of string-like crystals ([5]), nucleus formation ([6]), the growth on polyhedral crystals, polycrystalline precipitates, *etc.* These phenomena, however, cannot be discussed in this paper for lack of space and will be dealt with in a more detailed communication.

In conclusion, we may add that the growth of a crystal from a mixture of gases and solutions is subject to the same relationships as in electrocrystallization. This problem is also to be dealt with in a special communication.

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REFERENCES

[1] T. Erdey-Gruz u. M. Volmer, Z. phys. Chem., **157**, 165 (1931); К. Горбунова, Тр. 1-ой конф. по коррозии металлов АН СССР 1934г. стр. 144 (1934); V. Kohlschütter u. A. Torricelli, Z. Elektrochem., **38**, 213 (1932); J. Hoekstra, Rec. Trav. Chim. Pays-Bas., **50**, 339 (1931). [2] T. Erdey-Gruz u. M. Volmer, l. c. [3] К. Горбунова, Тр. 2-ой конф. по коррозии металлов 1940 г., II, стр. 142 (1943). [4] L. Prandtl, см., напр., Современные проблемы гидродинамики, **1**, 1943, стр. 64. [5] A. G. Samartzev, C. R. Acad. Sci. URSS, II, 478 (1935). [6] A. G. Samartzev and K. S. Evstropieva, Bull. Acad. Sci. URSS, sér. sci. nat., 603 (1934).

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OCÉANOLOGY

CHARACTERISTICS OF THE SPREADING OF ATLANTIC WATERS IN THE POLAR BASIN

By W. B. STOCKMANN

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Four years ago the author attempted to make a mathematical forecast of the characteristics of the temperature field in the direction of the movement of Atlantic waters spreading among the colder water masses of the Arctic seas and the Polar basin (1).

The author proceeded on the following simplifying assumptions: 1) The temperature field in the direction of the Atlantic water movement is stationary and is determined by mixing (of constant intensity) and by the advective heat transfer. 2) The velocity of movement of Atlantic waters is equal to that of the underlying polar water-masses, both in magnitude and direction. 3) The temperature on the surface of the Polar Ocean (or sea), usually ice-covered, is constant (freezing temperature). 4) The depth of the sea in the direction of the water movement is invariable. 5) Thermal isolation prevails at the bottom (the vertical heat current is zero). 6) Temperature distribution along a certain «initial» vertical is known.

Under these conditions we may easily obtain the solution of the differential equation

$$k\frac{\partial^2\vartheta}{\partial z^2}=u\frac{\partial\vartheta}{\partial x}=0 \qquad (1)$$

describing a two-dimensional stationary temperature field in the direction of the movement of Atlantic waters. The solution of equation (1) is

$$\vartheta=\vartheta_0+\frac{2}{h}\sum_{n=0}^{\infty}e^{-\frac{k}{u}\left[\frac{(2n+1)\pi}{2h}\right]^2 x}\sin\frac{(2n+1)\pi z}{2h}\int_0^h\vartheta(z)\sin\frac{(2n+1)\pi z}{2h}dz \qquad (2)$$

In formulae (1) and (2) ϑ is temperature; u is constant value of the current velocity in the direction of the horizontal axis x; k is constant value of the coefficient of turbulent mixing; h is depth of the sea; $\vartheta(z)$ is function describing the distribution of temperature along a certain initial vertical ($x=0$).

Since his aim at the time was to construct a theoretical model of the temperature field, the author selected the following arbitrary law for the initial distribution of temperature

$$\vartheta(z)=10\,(e^{-z}-e^{-2z})$$

This function satisfactorily describes the main features of vertical temperature distribution in the region of inflow of Atlantic waters to the Polar basin (North of Spitzbergen).

A model of the temperature field corresponding to some arbitrary values of k, v, z and x is shown in Fig. 1. Fig. 2 presents curves of temperature distribution along the verticals: I, $x = 0$; II, $x = 0.25$ and III, $x = 0.375$, according to the data of Fig. 1.

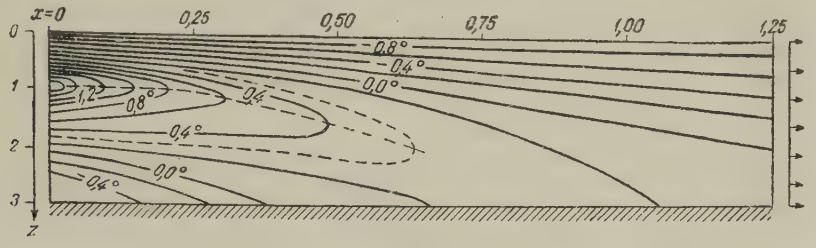

Fig. 1

On examining Figs. 1 and 2 one may easily notice the typical characteristics of the hypothetical temperature field in the direction of the movement of Atlantic waters. They consist in a gradual downward shifting of the temperature maximum, as the Atlantic waters penetrate deeper into the Polar basin. Moreover, with increas-

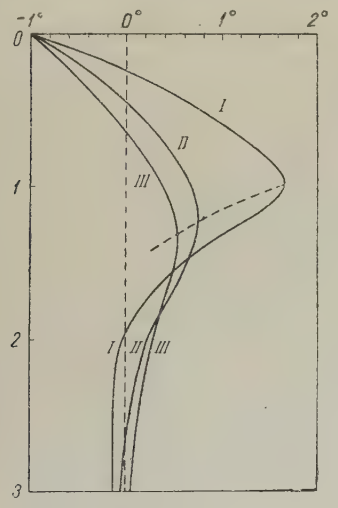

Fig. 2

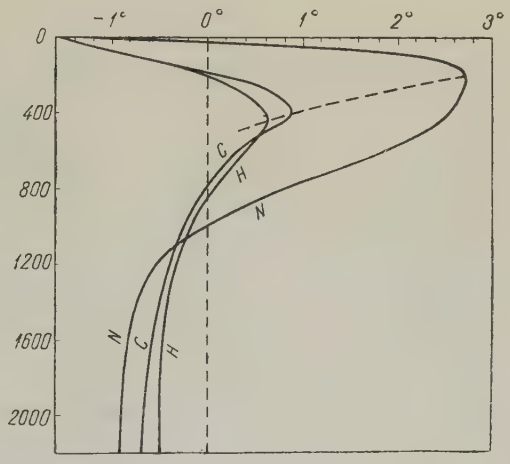

Fig. 3

ing distance from the initial vertical, the lowering of temperature in the core of Atlantic waters goes parallel with a rise of temperature in the bottom layers. As is seen from Fig. 1, the temperature in the bottom layers reaches its maximum at some distance from the initial vertical; then it falls off until the Atlantic waters become completely «dissolved» in the surrounding colder waters of the Polar basin.

Unfortunately, four years ago the author had no data at his disposal which could have confirmed or refuted the predicted peculiarities in the spread of Atlantic waters. At the present time, however, with the hydrographical observations of the «Sedov» drift worked up and with temperature measurements on hand taken on board the airplane H-169 in the region of the «Inaccessibility Pole» in 1941, this verification has become possible. In carrying it out the author compared the vertical temperature distribution on three points fairly distant from one another, located approximately in the direction of the Atlantic water movement and corresponding to the stations: No. 2 «Nautilus» designated by the letter N and with coordinates $\varphi = 81°59'$ N, $\lambda = 17°30'$ E; No. 7 «Sedov» designated C with coordinates $\varphi = 85°03'$ N, $\lambda = 131°48'$ E; and No. 1 of the airplane H-169 designated by H with coordinates $\varphi = 81°38'$ N, $\lambda = 179°02'$ W. Fig. 3 shows the curves of vertical temperature distribution on the above stations, and Fig. 4, the isotherms in the plane of the section selected.

Comparing Figs. 3 and 4 with Figs. 1 and 2, we see that the actual temperature distribution in the direction of the movement of Atlantic waters possesses all the characteristics of the theoretical scheme. Thus, in Fig. 3, and likewise in Fig. 2, the temperature of the lower layers of the section (1100—2200 m) regularly increases with the distance from the initial vertical (station N), whereas the absolute value of the temperature maximum in the upper layer declines. The temperature maximum shows a constant downward shifting, a circumstance which also agrees with the previous findings, and the distribution of isotherms in Fig. 4 bears a close

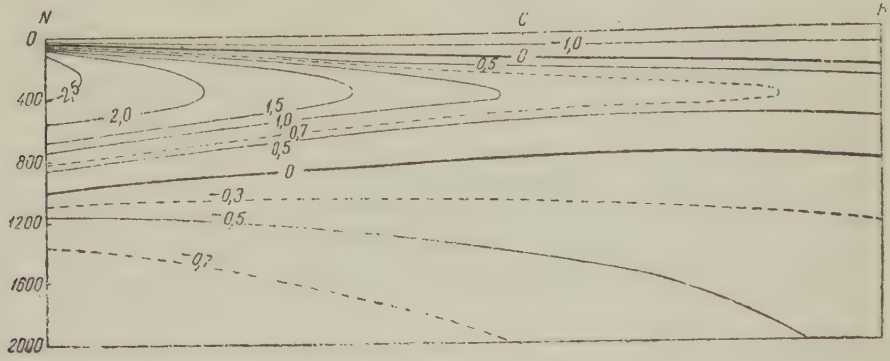

Fig. 4.

resemblance to the theoretical scheme (Fig. 1) within the distance from $x = 0$ to $x = 0.25$.

Thus, our theoretical forecast is not merely confirmed by observations*, but it also provides a theoretical explanation for the remarkable features revealed in the spreading of Atlantic waters in the Polar basin. Furthermore, a comparison of Figs. 1 and 4 shows convincingly that the deeper waters of the Polar basin, which, according to Nansen, penetrate there from the Greenland Sea, move with the Atlantic waters nearly in the same direction. It should also be emphasized that the actually observed fall of the temperature maximum in Atlantic waters is not necessarily the result of a sinking of the Atlantic layer, as was supposed formerly, but may well be ascribed to vertical intermixing.

Naturally enough, there can hardly be a quantitative agreement between Figs. 1, 2, 3 and 4, not only because arbitrary values were used in the construction of the theoretical scheme, but also because such factors as the variation of the mixing coefficient and of the velocity of the current were not taken into account.

In principle, the general picture of the phenomenon will not change if we take them into consideration, and we shall then be able to proceed from a schematic representation of the process to a detailed computation of the temperature field. This will be done by the author elsewhere.

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2.VI.1944.

REFERENCES

[1] В. Б. Штокман. Проблемы Арктики, 12 (1940).

* The theoretical scheme of the phenomenon is also confirmed by the results reported by P. P. Shirshov.

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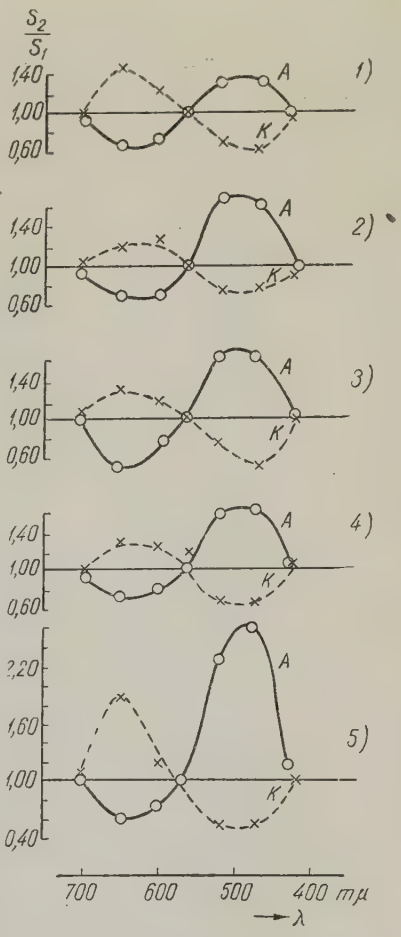
BIOPHYSICS

COLOUR VISION AS AFFECTED BY ELECTROTONUS

By S. V. KRAVKOV and L. P. GALOCHKINA

(Communicated by S. I. Vavilov, Member of the Academy, 1.XII.1944)

Our object has been to study how a constant current, while it is allowed to pass for some time through the eye, affects colour sensitivity. In the experiments a dark-adapted eye was investigated for its foveal colour sensitivity to different monochromatic rays of the spectrum. The electric current was conveyed to the eye by means of silver electrodes. One of the electrodes—a plate — was placed in the hand of the person tested, and the other electrode was pressed either to the eye-lids (the eye being kept open) or (in control tests) to the temples, near the corner of the eye. The current strength was always 0.02 mA. The visibility thresholds for the different monochromatic rays were determined during 50—60 minutes of dark adaptation. Ater about 30 minutes of adaptation, when the foveal sensitivity had reached a constant level, the current was switched on and allowed to act upon the eye for 4 minutes. While the current was in action, two or three determinations of the threshold were made. For the index we took the ratio of colour sensitivities S_2/S_1, where S_1 is the colour sensitivity of the eye before switching on the current, and S_2, 3 minutes after.



The experiments were run of five persons with normal colour vision (see Nos. 1—5 — on the figure). The results obtained with different persons coincide perfectly. In the adjoined figure the abscissae are wave-lengths in mμ of the light for which the threshold of visibility was measured. The ordinates represent the S_2/S_1 ratio. The letters A and K indicate the curves bringing out the results of the experiments in which the electrode applied to the eye was the anode (anodic electrotonus) and cathode (cathodic electrotonus), respectively.

It is evident that in the case of anodic electrotonus the sensitivity to green-blue rays increases and the sensitivity to orange-red rays declines. The relaton is reverse in the case of cathodic electrotonus: there is an increase in the sensitivity to orange-red rays and a decrease in the sensitivity to green-blue rays.

In both cases there appears to be three regions of the spectrum for which there was no change in the sensitivity of the eye. These are the end-parts of the spectrum and the yellow part (in the vicinity of the 570 mμ).

After the current was interrupted, the colour sensitivity of the eye displayed alterations inverse to those which were observed during the passage of the current.

These facts may, in our opinion, be explained by those migrations of the ions which take place under the influence of a constant current; in the first place, by the well-known circumstance that the relative concentration of Ca ions on the anode undergoes an increase, whereas on the cathode it is the relative concentration of potassium ions that increases. Now, it is known to physiologists that the effect of calcium is often similar to that produced by the stimulation of the sympathetic nerve, and the effect of potassium is analogous to the action of the parasympathetic nerve [1].

Therefore the existence of a definite relation between colour vision and ionic changes, as revealed in our experiments, is another evidence of the dependence of our colour vision upon the vegetative nervous system [2].

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1.XII. 1944.

REFERENCES

[1] S. G. Zondek, Bioch. Z., **132**, 362 (1922); Klin. Wochenschr., No. 9,382 (1923); Bioch. Z., **132**, 362 (1922); J. Tinel, Le système nerveux végétatif, Paris, 813, 1927.
[2] S. V. Kravkov, Acta Medica URSS, **2**, 461 (1939); S. V. Kravkov, J. Opt. Soc. Am., **31**, 335 (1941).

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TECHNICAL PHYSICS

ELECTRIC PRECISION MEASUREMENTS BY MEANS OF INCANDESCENT LAMP USED AS INDICATOR OF CURRENT EQUALITY IN TWO CIRCUITS

By A. I. FÜRSTENBERG

(Communicated by S. I. Vavilov, Member of the Academy, 7.VIII.1944)

In an incandescent lamp with a metal filament the appearance of a barely visible ligth emission is a very fine criterion for visual estimation of the «heating» current.

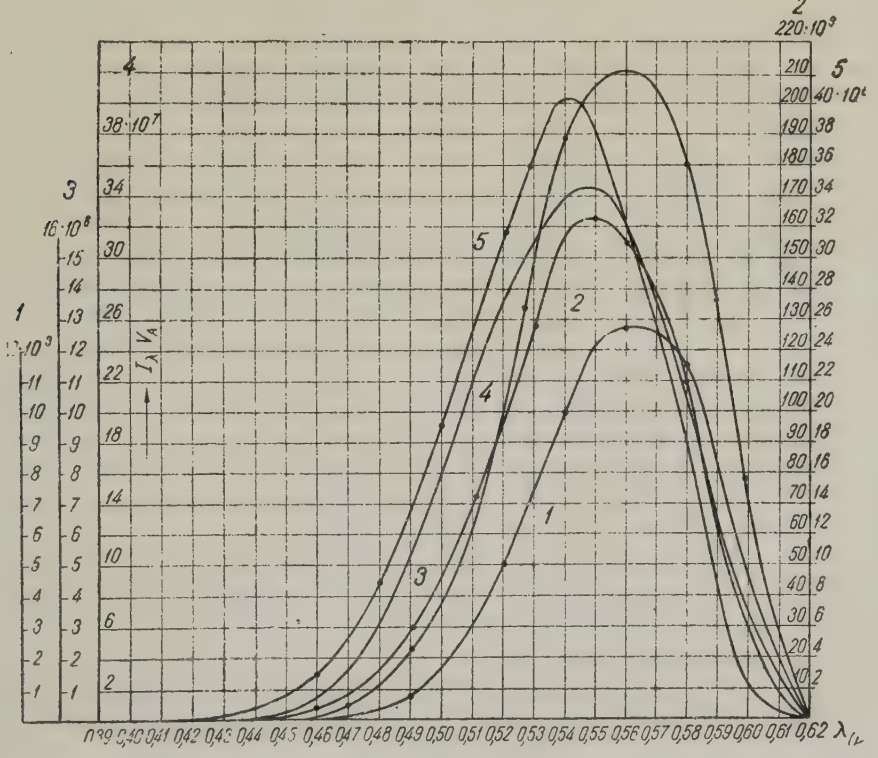

Fig. 1. *1*, T=900° K; *2*, T=1000° K; *3*, T=1200° K; *4*, T=1400° K; *5*, T=1600° K

A circumstance of undoubtful interest for metrology is that under conditions of a vanishingly small light emission by the filament slight increments in the current passing through the lamp have a considerable effect upon brightness.

Fig. 1 represents a family of curves $I_\lambda V_\lambda = f(\lambda)$ for different values of «black-body» temperature (L_λ is the monochromatic intensity of radiation; V_λ is the coefficient of relative visibility). The areas of these curves are proportional to the values of brightness for a given temperature.

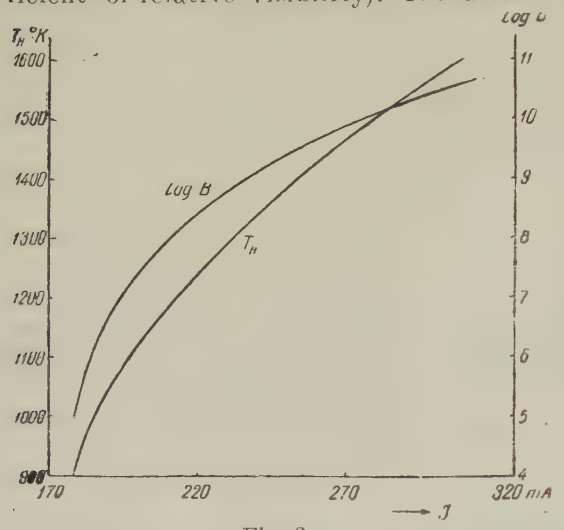

Fig. 2

Fig. 2 brings out the experimentally observed dependence between the temperature T_H of the central portion of the filament in a 2.5 volt lamp and the intensity of the current. Fig. 1 and the T_H-curve of Fig. 2 were used as a basis for plotting the dependence of the filament brightness upon current strength (the log B curve, Fig. 2). The latter shows that in the neighbourhood of the «lighting» threshold ($T =$ 900—1000°K) as slight an increase of the current strength as ± 0.5 per cent results in a ± 50 per cent increase of brightness.

What makes it difficult in the first place to measure the absolute values of current strength and voltage by fixing visually the start of light emission, is that the threshold value of the latter is subject to considerable individual fluctuations and depends, moreover, upon a variety of factors which cannot all be taken into account.

The experiments performed by the author have shown that with one orbserver after 10 minutes of dark adaptation, the accuracy of determinations of the current intensity is ± 0.01 per cent. Some hours after, however, the reproducibility of the estimates becomes rather low. The widest scattering of mean estimates of the current strength or the 12 observers experimented upon amounted to 0.95 per cent of the measured magnitude.

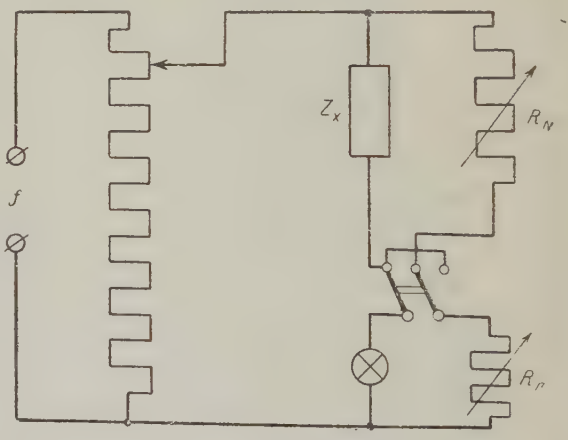

Fig. 3

By far greater possibilities opens up the comparison method, which consists in determining by means of the lamp the equality of currents in two circuits (Fig. 3). The circuits are connected in parallel and are made to carry currents of equal strength. One of the currents is that to be measured, the other is the standard current. By means of switch the lamp or its equivalent can either be connected to the circuit whose impedance is measured, or to the circuit of standard resistance. The initial position of the switch is to the left. By controlling the voltage the lamp is made to emit a barely visible light. Then the switch is turned to the right, and by regulating the standard resistance the filament of the lamp is again adjusted to bare visibility. When the currents in the right and left branches are equal, we obviously have the relation

$$\overline{Z}_x + \overline{R}_{Л} = R_N + R_{Л} \qquad (1)$$

Here Z_x is the measured total resistance (in ohms), $R_Л$ is the resistance of the lamp under working conditions (in ohms), R_N is the resistance of the box (in ohms).

In order to determine the subjective error involved in such relative estimation of brightness when the two currents approach equality, an experiment was arranged with 10 observers chosen at random. Repeated measurements of the same strictly constant value (the 100-ohm standard resistance of the «Etalon» plant) were made. Due care was exercised to separate the purely subjective error from deviations caused by other factors. The lamp was placed in a cylinder painted dead-black and provided with a shield protecting the eye from the influence of external light. It was found that the probable error in the arithmetical mean derived from five measurements of the same current strength never exceeded ± 0.03 per cent for any observer.

Let us dwell in brief upon the errors involved in the measurement of capacitance and inductance, when compared by the described method with active resistance and frequency. The cases where frequency, impedance, reactance, active resistance, *etc.* are measured can be dealt with in the same way as described below. Let us start with the measurement of the capacitance.

If there are no losses in the condensor, the relation (1) gives

$$C_x = \frac{1}{\omega \sqrt{R_N^2 + 2R_N R_Л}} \qquad (2)$$

Here C_x is the measured capacity in farads; ω is the circular frequency of the current in radians per second. Let us evaluate the error due to the presence of active losses in the measured capacitance. If the real condensor C_x is replaced by an ideal one C_x' connected in parallel with the active resistance ρ, the angle of losses being $\delta = \operatorname{arctg} \omega C_x' \rho$, we get

$$\sqrt{\left(\frac{1}{\omega C_x'}\right)^2 + (R_Л + \rho)^2} = R_N + R_Л$$

After simple computations we shall have

$$\delta_\rho = \frac{\Delta C_x}{C_x} \cong \frac{R_Л}{R_N} \operatorname{tg} \delta = 100 \frac{R_Л}{R_N} \operatorname{tg} \delta \, (\%) \qquad (3)$$

The variation in the resistance of the lamp due to the inconstancy of the «lighting» current gives an error $\delta_{R_Л}$.

Suppose the resistance of the lamp to have changed from $R_Л$ to $R_Л + \Delta R_Л$. Then expression (2) will take the form

$$C_x + \Delta C_x = \frac{1}{\omega \sqrt{R_N^2 + 2R_N (R_Л + \Delta R_Л)}} \qquad (4)$$

After the radicals in (2) and (4) are developed into a series and all terms higher than the second are neglected, the relative error can be computed as follows:

$$\delta_{R_Л} = \frac{\Delta C_x}{C_x} \cong \frac{\Delta R_Л}{R_N} 100\% \qquad (5)$$

In order to reduce this error to a minimum, the measurement of the resistance of the lamp under given conditions must be made directly after the circuit is brought into balance.

The frequency fluctuation of the source leads, as it follows from expression (2), to similar (except for the reversal of sign) errors in the capacitance estimates

$$\delta_f = \frac{\pm \Delta f}{f} = \frac{\pm \Delta C_x}{C_x} \qquad (6)$$

By using a tuning-fork controlled oscillator as a source of energy with an amplifier fed from a battery or from a voltage stabilizer, one can without special difficulties obtain $\Delta f \approx (1 \div 3) \cdot 10^{-3}$ per cent for a fairly constant amplitude of voltage, which is also a very essential circumstance.

The influence of the harmonics of the energy source voltage curve can be taken into account by means of the following formula:

$$\delta_{\text{harm}} = \frac{\Delta C_x}{C_x}\left[\sqrt{\frac{1+4\left(\frac{U_2}{U_1}\right)^2+9\left(\frac{U_3}{U_1}\right)^2+\dots}{1+\left(\frac{U_2}{U_1}\right)^2+\left(\frac{U_3}{U_1}\right)^2+\dots}}-1\right]100\% \qquad (7)$$

Here U_1 is the amplitude of voltage of the main frequency. U_n is the voltage amplitude of the number n harmonic.

By making use of tuned circuits in the amplifier, one can easily obtain $\delta_{\text{harm}} \approx (2 \div 5) \cdot 10^{-3}$ per cent.

The error due to the resistance box in using correction tables must be estimated as $\pm$ 0.01 per cent. The residual reactance of the modern resistance boxes is so small (the time constant $\tau \approx 10^{-7} \div 10^{-8}$ sec.) that it can be neglected in a range of up to 10 kilocycles. Thus, an accuracy of about $\pm$ (0.05 — 0.1) per cent can be obtained in capacity measurements made by a rather simple procedure.

The measurement of inductance by this method has much in common with the capacity measurement.

Upon bringing the circuit into balance as described above, the following relation takes place

$$L_f = \frac{1}{\omega}\sqrt{R_N^2 + 2R_N R_Л - R_f^2 - 2R_f R_Л} \qquad (8)$$

Here L_f is the inductance of the coil at a frequency f; R_f is the active resistance of the coil at this frequency. Otherwise the notation remains the same.

In accordance with what has been said above the reason for the appearance of the error $\delta_{R_Л}$ lies in the fluctuations of the resistance of the lamp. If the resistance of the lamp has suffered an increment of $\Delta R_Л$, then

$$\delta_{R_Л} = \frac{\Delta L_f}{L_f} \cong \frac{\Delta R_Л}{R_N} 100\% \qquad (9)$$

In measuring small inductances with a high precision, lamps with a low ohmic resistance are to be used in order to decrease the value of $\delta_{R_Л}$.

The influence of the harmonics entering into the voltage feeding the circuit is here much less than in the case of capacity measurements, *viz.*

$$\delta_{\text{harm}} = \frac{\Delta L_x}{L_x}\left[\sqrt{\frac{1+\left(\frac{U_2}{U_1}\right)^2+\left(\frac{U_3}{U_1}\right)^2+\dots}{1+\frac{1}{4}\left(\frac{U_2}{U_1}\right)^2+\frac{1}{9}\left(\frac{U_3}{U_1}\right)^2+\dots}}-1\right]100\% \qquad (10)$$

The other possible sources of error have been considered above. The total error of measurement can be reduced to about (0.05 ÷ 0.1) per cent.

This method is universal in the sense that it can be used as an absolute method of high precision and as a simple absolute method of measurement of medium accuracy, and finally, as a method of comparison against standard measures of capacitance and inductance with a precision sufficient for technical purposes.

In conclusion the author desires to convey his heartiest thanks to S. I. Vavilov for interest in this work and valuable advice. Thanks are also due to G. S. Landsberg and L. A. Tummerman.

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HEAT ENGINEERING

ON A MODIFICATION OF FORCHHEIMER'S PROBLEM

By I. A. CHARNY

(Communicated by N. E. Kochin, Member of the Academy, 30.VIII.1944)

1. The problem dealt with here is to determine heat losses from a unit length of a tube buried in the ground, when the coefficient of heat transfer k from the surface of the ground to the air is a finite value. Forchheimer has

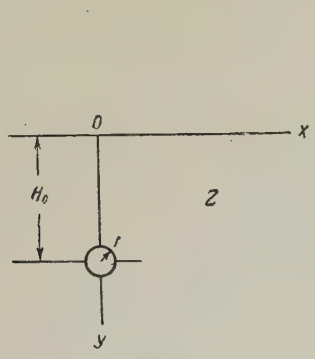

Fig. 1

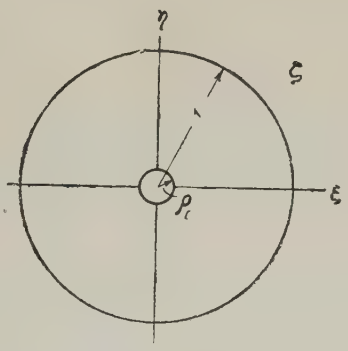

Fig. 2

solved there a similar problem for the case where the coefficient of heat transfer is not given, but the temperature of the ground surface is put at zero instead.

The temperature of the surface of the ground adjoining the tube T_0 is supposed to be known. The temperature of the air is set at zero. The thermal conductivity of the ground is denoted by λ. The distribution of temperature is a settled one. The depth and radius of the tube are H_0 and r, respectively (Fig. 1).

On these assumptions the problem is reduced to the solution of the equation $\Delta T = 0$ under the boundary conditions:

$$\left.\begin{aligned} &T = T_0 \text{ at the surface of the tube} \\ &\text{for } y = 0 \qquad kT = \lambda \frac{\partial T}{\partial y} \end{aligned}\right\} \tag{1}$$

2. Let us introduce the function

$$W(z) = \Phi + i\Psi \tag{2}$$

where

$$\Phi = \lambda T \tag{3}$$

Ψ is the stream function.

Making use of the equations of Cauchy-Rieman

$$\frac{dW}{dz} = \frac{\partial \Phi}{\partial x} + i\frac{\partial \Psi}{\partial x} = \frac{\partial \Phi}{\partial x} - i\frac{\partial \Phi}{\partial y}$$

we can write the boundary conditions (1) as follows

$$\frac{k}{\lambda} R(W) = -I\left(\frac{dW}{dz}\right) \text{ for } y = 0 \tag{4}$$

$$R(W) = \lambda T_0 = \Phi_0 \text{ at the surface of the tube} \tag{5}$$

Thereby the problem is reduced to the determination of a function of the complex variable $W(z)$ satisfying (4), (5) and vanishing at infinity.

3. By means of the function

$$\zeta = \frac{z - iH}{z + iH} \tag{6}$$

we map the ground temperature region on the plane z into a circular ring of the plane ζ with radii $\rho_0 < 1$ and $\rho = 1$, so as to make the ox-axis pass into a circumference of radius $\rho = 1$, and the cross-section of the tube, into a circle of radius $\rho = \rho_0$.

The constants H and ρ_0 will be determined below. On the planes ζ (4) and (5) will have the form:

$$|\zeta| = 1: \quad \frac{k}{\lambda} R(W) = -I\left(\frac{dW}{d\zeta} \Big/ \frac{dz}{d\zeta}\right) = -I\left[\frac{(1-\zeta)^2}{2iH}\frac{dW}{d\zeta}\right] \tag{4.1}$$

$$|\zeta| = \rho_0: \quad R(W) = \Phi_0 \tag{5.1}$$

Within the ring W is expendable into a series

$$W = A \ln \zeta + \sum_{-\infty}^{\infty} a_n \zeta^n \tag{7}$$

where

$$\zeta = \rho e^{i\theta}, \quad a_n = \alpha_n + i\beta_n \tag{7.1}$$

From (7) we obtain

$$|\zeta| = \rho_0: \quad R(W) = A \ln \rho_0 + \alpha_0 + \sum_{n=1}^{\infty} (\alpha_n \rho_0^n + \alpha_{-n}\rho_0^{-n}(\cos n\theta - \\ - (\beta_n \rho_0^n - \beta_{-n}\rho_0^{-n}) \sin n\theta \tag{8}$$

$$|\zeta| = 1: \quad -I\left[\frac{(1-\zeta)^2}{2iH}\frac{dW}{d\zeta}\right] = \frac{1}{2H}\left\{2A(\cos\theta - 1) + \right.$$

$$+ \sum_{n=1}^{\infty} n(\alpha_n - \alpha_{-n})[\cos(n-1)\theta - 2\cos n\theta + \cos(n+1)\theta] -$$

$$\left. - \sum_{n=1}^{\infty} n(\beta_n + \beta_{-n})[\sin(n-1)\theta - 2\sin n\theta + \sin(n+1)\theta]\right\} \tag{9}$$

From (6), (7) and the conditions at infinity we get

$$z = \infty, \quad \zeta = 1, \quad \sum_{-\infty}^{\infty} a_n = 0 \tag{10}$$

Comparing the coefficients, we find in virtue of (4.1) and (5.1) that

$$\left.\begin{aligned} &\alpha_0 + A \ln \rho_0 = \Phi_0; \quad \frac{1}{2H}(-2A + \alpha_1 - \alpha_{-1}) = \frac{k\alpha_0}{\lambda}; \\ &\alpha_n \rho_0^n + \alpha_{-n} \rho_0^{-n} = 0; \quad \beta_n = 0 \end{aligned}\right\} \tag{11}$$

$$\frac{1}{2H}[2A + 2(\alpha_2 - \alpha_{-2}) - 2(\alpha_1 - \alpha_{-1})] = \frac{k}{\lambda}(\alpha_1 + \alpha_{-1}) \tag{12}$$

. . . . . .

$$\frac{1}{2H}[(n+1)(\alpha_{n+1} - \alpha_{-(n+1)}) - 2n(\alpha_n - \alpha_{-n}) + (n-1)(\alpha_{n-1} - \alpha_{-(n-1)})] =$$

$$= \frac{k}{\lambda}(\alpha_n + \alpha_{-n}) \tag{13}$$

From (10) we have

$$\alpha_0 + \sum_{-\infty}^{\infty} \alpha_n = 0 \tag{14}$$

The coefficients α_n and A are determined as follows: from (11) and (12) the coefficients α_0, α_1, α_{-1}, α_2, α_{-2} may be expressed through A as

$$\left.\begin{aligned} &\alpha_0 = \Phi_0 - A \ln \rho_0; \quad \alpha_1 = 2\frac{\beta\Phi_0 + (1 - \beta \ln \rho_0) A}{1 + \rho_0^2}; \quad \alpha_{-1} = -\alpha_1 \rho_0^2; \\ &\alpha_{-2} = -\alpha_2 \rho_0^4; \quad \beta = \frac{kH}{\lambda}; \quad \alpha_2 = \frac{1}{2}\frac{2\beta\alpha_0 + [(2\beta+1) - (2\beta-1)\rho_0^2]\alpha_1}{1 + \rho_0^4} \end{aligned}\right\} \tag{15}$$

The subsequent coefficients are computed by the following recurrent formula obtained from (13)

$$\alpha_{n+1} = \frac{2[(\beta+n)\alpha_n + (\beta-n)\alpha_{-n}] - (n-1)(\alpha_{n-1} - \alpha_{-(n-1)})}{(n+1)(1+\rho_0^{2(n+1)})} \tag{16}$$

Thus, all the coefficients α_n can be expressed linearly through A. Substituting in (14) the values of α_n, we obtain the equation for determining A. The loss of heat from a unit length of the tube is determined from the formula

$$Q = -2\pi A \tag{17}$$

The relation between H_0 and radius r (Fig. 1) with H and ρ_0 is easily established from (6). After computation we obtain

$$H = H_0 \frac{1 - \rho_0^2}{1 + \rho_0^2} \tag{18}$$

$$\rho_0 = \frac{H_0}{r}\left(1 - \sqrt{1 - \frac{r^2}{H_0^2}}\right) \tag{19}$$

If r/H_0 is small, which is usually the case, then

$$\rho_0 \approx \frac{r}{2H_0}, \qquad H_0 \approx H \tag{19.1}$$

The variables ρ and θ are connected with x, y by the relations:

$$\rho^2 = \frac{x^2 + (y-H)^2}{x^2 + (y+H)^2}; \quad \theta = \operatorname{arctg}\frac{y-H}{x} - \operatorname{arctg}\frac{y+H}{x} \tag{20}$$

4. In order to represent the solution in the form of an infinite series (7) one has to do a great amount of computation in determining the coefficients, and afterwards to investigate the character of the convergence. Let us proceed therefore in the following way. We reduce (7) to a single term and see, how much the boundary condition (4) is thereby distorted.

From (9) and (5) we instead of (4) obtain

$$|\zeta|=1: \quad \frac{k}{\lambda}\,\mathrm{R}\,(W)+\mathrm{I}\left[\frac{(1-\zeta)^2}{2iH}\frac{dW}{d\zeta}\right]=-\frac{\alpha_1-\alpha_{-1}}{2H}\cos 2\vartheta \tag{21}$$

Putting $\alpha_n=\alpha_{-n}=0$ for $n>1$, the remaining coefficients can, according to 11 and (14), be determined from the equations:

$$\left.\begin{aligned} &\alpha_0+A\ln\rho_0=\Phi_0; \quad \frac{1}{2H}(-2A+\alpha_1-\alpha_{-1})=\frac{k\alpha_0}{\lambda}; \\ &\alpha_1\rho_0+\frac{\alpha_{-1}}{\rho_0}=0; \quad \alpha_0+\alpha_1+\alpha_{-1}=0; \quad \beta_1=\beta_{-1}=0 \end{aligned}\right\} \tag{22}$$

The solution of (22) gives

$$A=\Phi_0\,\frac{1+\frac{1+\rho_0^2}{2(1-\rho_0^2)}\frac{\lambda}{kH}}{\left[1+\frac{1+\rho_0^2}{2(1-\rho_0^2)}\frac{\lambda}{kH}\right]\ln\rho_0-\frac{\lambda}{kH}} \tag{23}$$

$$\left.\begin{aligned} \alpha_0&=-\Phi_0\,\frac{\lambda/kH}{\left[1+\frac{1+\rho_0^2}{2(1-\rho_0^2)}\frac{\lambda}{kH}\right]\ln\rho_0-\frac{\lambda}{kH}} \\ \alpha_1&=\Phi_0\,\frac{\lambda/kH}{\left\{\left[1+\frac{1+\rho_0^2}{2(1-\rho_0^2)}\frac{\lambda}{kH}\right]\ln\rho_0-\frac{\lambda}{kH}\right\}(1-\rho_0^2)} \\ \alpha_{-1}&=-\Phi_0\,\frac{\frac{\lambda}{kH}\rho_0^2}{\left\{\left[1+\frac{1+\rho_0^2}{2(1-\rho_0^2)}\frac{\lambda}{kH}\right]\ln\rho_0-\frac{\lambda}{kH}\right\}(1-\rho_0^2)} \end{aligned}\right\} \tag{24}$$

whence

$$\frac{\alpha_1-\alpha_{-1}}{2H}=\Phi_0\,\frac{\frac{1}{2}\frac{\lambda}{kH^2}\frac{1+\rho_0^2}{1-\rho_0^2}}{\left[1+\frac{1+\rho_0^2}{2(1-\rho_0^2)}\frac{\lambda}{kH}\right]\ln\rho_0-\frac{\lambda}{kH}} \tag{25}$$

It may be seen from (5), (21) and (25) that the boundary conditions are thus distorted by our approximation: the coefficient k/λ is no longer a constant, but varies within the limits

$$\frac{k}{\lambda}\left[1\pm\frac{1}{2}\left(\frac{\lambda}{kH}\right)^2\frac{1+\rho_0^2}{1-\rho_0^2}\right] \tag{26}$$

For practically important cases λ/kH does not exceed $0.1-0.2$, and this variation can be neglected. Then (17), (8), (23) and (24) can be used in determining the heat losses and the temperature field.

It has been shown by Chernikin [1] that the computation of heat losses by the above method yields results that differ little from those obtained by means of the formulae of Forchheimer, Shubin and others.

On the other hand, computations of temperature fields made by using different formulae give by far more divergent results. The closest agreement with the experiment is obtained, according to Chernikin, by employing the method suggested in the present paper.

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REFERENCES

[1] В. И. Черникин, Диссертация, Моск. Нефт. институт им. Губкина, 1944.

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CHEMISTRY

DECAHYDRONAPHTHALENE PEROXIDE

By K. I. IVANOV and V. K. SAVINOVA

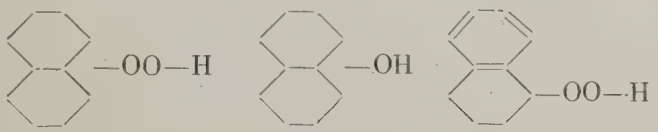
(Communicated by A. N. Nesmeyanov, Member of the Academy, 21.VIII.1944)

The oxidation of decahydronaphthalene (decaline) in the liquid phase by molecular oxygen starts and proceeds with considerable velocity at a much higher temperature than tetraline, this making it difficult to trace the intermediate reactions and to isolate the peroxide formed.

In order to substitute thermal activation of the process by some other form of activation, decaline was subjected to the action of oxygen under the simultaneous action of ultra-violet rays (the method used by us for the first time in the preparation of isopropyl ether peroxide). In this way it proved possible to start the oxidation process at a comparatively low temperature and, by stopping it at the stage of the formation of the peroxide, to solate the latter in a pure state and to investigate it in detail *.

Decaline peroxide has the formula $C_{10}H_{18}O_2$. It is a solid crystallizing in long thin needles (from petroleum ether) with a melting point of 96.5°. The peroxide dissolves readily in diethyl ether, benzene, CCl_4, methyl and ethyl alcohol, somewhat worse in cold petroleum ether and is insoluble in water. In the flame of the Bunsen burner it burns evenly, without explosions. It liberates iodine but slowly from an HI solution, is not reduced completely when heated with an aqueous solution of sodium sulphite, reacts with concentrated NaOH solutions (aqueous-alcoholic) only at high temperatures forming the sodium salt, is not condensed quantitatively with HCHO (forming hydroxymethyl-peroxide). Decaline peroxide is stable under usual conditions of storage. A sample that had been standing in the laboratory for 2 years had almost not changed its properties (including the active oxygen content).

A study of the structure and properties of decaline peroxide showed that it belongs to the class of hydroperoxides having a —OO—H group at the tertiary carbon atom in the decaline molecule (γ-position) (I)**



I II III

* Decaline peroxide was first isolated and investigated by us in 1940 and then the work was interrupted. The structure of the peroxide was finally determined in 1944.

** The fact that a small amount of a peroxide with very similar constants was isolated afterwards in our laboratory (S. Krein and A. Mikhelson) from among the products of a short oxidation of decaline with oxygen under pressure at 120° (without using ultra-violet rays) shows that the action of ultra-violet rays does not change the course of the oxidation of decaline from that of the process which takes place under the influence of only thermal activation.

This structure is proved by the following:

1. Reduction of the peroxide yields 9-hydroxydecaline, m. p. 54.7° (trans-form) (II) which may be identified in the form of its crystalline chromic ester, peculiar only to tertiary transdecalol (α and β-decalols, being secondary alcohols, give no addition products with CrO_3 but are oxidized by him into the corresponding ketones; cis-γ-decalol forms a liquid ester with CrO_3).

2. The peroxide is capable of forming salts with the hydroxides of alkaline metals.

The compound, having the structure given above (I) can exist in two stereoisomeric forms.

The peroxide having the properties described here was obtained by photo-oxidation of a mixture of both decaline isomers purified from other admixtures, and by photo-oxidation of pure trans-decaline* (b. p. 95—95.5° at 50 mm Hg, d_4^{20} 0.869, n_D^{20} 1.4697).

The latter circumstance, as well as the fact that reduction of the hydroperoxide by different methods gave in all cases only trans-γ-decalol, the yields being almost theoretical, lead to the conclusion that the decaline hydroperoxide obtained was the transisomer.

The relatively high thermal stability, already mentioned above as well as the chemical inertness of decaline hydroperoxide is worthy of attention as making the latter different from all known hydroperoxides having hydrocarbon radicals whith an different structure. This inertness which can be also seen from the absence of the faculty to form alkaline salts rapidly and quantitatively, hinders the isolation of decaline peroxide from the oxidized hydrocarbon by extracting it with an aqueous NaOH solution, which is easily achieved in the case of tetralin hydroperoxide (III) [1]. These properties are determined by the structure of the peroxide obtained and particularly:

a) by the absence in its molecule of aromatic nuclei and of active double bonds; the proximity of the latter to the hydroperoxide group evidently stipulates increased reaction capacity;

b) the greater stability of trans-decaline and its derivatives in comparison with the corresponding cis-compounds, this being due to steric factors.

Preparation of Decaline Peroxide. After repeated fractionation in order to eliminate most of the admixtures, commercial decaline was treated with 3—4 volumes of 95 per cent sulphuric acid (in several portions until the acid layer stopped darkening) and a 10 per cent silica gel, was washed with a 5 per cent NaOH solution and then with water, then dried over $CaCl_2$ and distilled over sodium. The product purified by this method was a mixture of cis-(n_D^{20} 1.4806[2]) and of trans- (n_D^{20} 1.4695 [2]) decaline, with the latter prevailing. Thus, one of the characteristic samples boiled at 184—187°/741 mm Hg and had n_D^{20} 1.4708 and d_4^{20} 0.873.

The purified decaline (200 g) was oxidized in a quartz flask fitted with a reflux condenser by bubbling oxygen through the hydrocarbon at a rate of 5 cm³ per minute under ultra-violet radiation (the region of great wave-lengths) from a mercury lamp in the course of 200 hrs at 70°. After distilling off the residual decaline from the reaction product (in the quartz apparatus at a pression of 1—2 mm Hg over a water bath in a current of dry nitrogen) there remained a transparent, very pale yellow liquid from which crystals of the peroxide precipitated upon standing and more rapidly under cooling**. The white crystalline mass was filtered under suction on a Schott filter and recrystallized three times from its petroleum ether.

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* The identity of the peroxide preparations obtained in both cases was confirmed by the coincidence of physical-chemical constants, by there being no depression of the mixed melting point of the samples when mixed, and by the identity of the optical constants (refractive index, angle of optical axes).

** If no crystallization takes place, the peroxide can be isolated by distillation of the liquid residue in a vacuum (1—1.5 mm Hg).

Decaline peroxide was obtained in long prismatic needles, of the monoclinic system*. The yield was 1—1.5 per cent.

Refractive indices of the decaline peroxide crystals, measured by the immersion method were: $N_g = 1.570$; $N_m \approx N_p = 1.550$ (Na); angle of optical axes $2V = +12°$; angle of extinction 20°.

The melting point of pure decaline peroxide is 96.5°, the boiling point 103° C at 1.2 mm Hg; sp. weight $d_4 = 1.15$ (solid).

Analyses:

I 0.2410 g of substance: 0.6268 g CO_2; 0.2314 g H_2O
II 0.2206 g » » 0.5709 g CO_2; 0.2114 g H_2O
Found %: C 70.98; 70.58; H 10.74; 10.72
$C_{10}H_{18}O_2$: calculated %: C 70.59; H 10.59
0.0896 g of substance: 17.44 g of benzene; Δt 0.160°.
Found: M 163.7. $C_{10}H_{18}O_2$: calculated M 170.0

The content of active oxygen, determined by the stannometric method** corresponds to an expenditure of 19.55 cm³ of a 0.1 N $FeCl_3$ solution per 1 millimole of the peroxide; the calculated expenditure for $C_{10}H_{17}$—OO—H is 20.00 cm³ of 0.1 N $FeCl_3$ solution per 1 millimole.

The peroxide shows qualitative reactions for active oxygen (liberation of iodinen from a HI solution, colouring of titanic acid, and after treatment with formaldehyde in benzene solution and evaporation of the solvent it clarifies an alkaline solution of methylene blue.

Reduction of Decaline Peroxide with Hydrogen Iodide. 4 g of the peroxide were dissolved in 30 cm³ of glacial acetic acid. The solution was diluted with water to stable turbidity, and 15 g of KI were added. After standing for two days the liberated iodine was removed by using a normal solution of thiosulphate. The solution was neutralized with sodium carbonate and distilled with steam. The crystals of decalol were separated from the aqueous distillate and were dried first between two sheets of filter paper and then in a dessicator over H_2SO_4 (yield 3 g). After crystallization from petroleum ether the decalol had a melting point of 54.0°, and after subsequent sublimation, 54.7°. Further sublimation did not change this melting point. Extremely thin leaves were obtained, that sublimed very easily, had a strong odour of mint and gave no reaction for peroxidic oxygen.

0.1912 g of substance: 0.5456 g CO_2; 0.2140 g H_2O
Found %: C 77.82; H 12.52
$C_{10}H_{18}O$: calculated %: C 77.85; H 11.77
0.0908 g of cubstance; 17.65 g of benzene; Δt 0.172
Found M 152.0. $C_{10}H_{18}O$: calculated M 154.0.

Reduction of Decaline Peroxide with Zinc Dust. To 0.47 g of the peroxide dissolved in 8 cm³ of glacial acetic acid 8 cm³ of water were added gradually. After the addition of 0.6 g of zinc dust the mixture was refluxed for 1 hour and then distilled with steam. After neutralization the crystalline substance was separated from the distillate using the method described above. The isolated crystalline substance had properties similar to those of the reduction product of the peroxide with hydrogen iodide. After sublimation the melting point was 54.5°. A mixture of this substance with trans-decalol-9 (m. p. 54.7°) obtained through the reduction of the peroxide with HI, had a melting point 54.7°.

Trans-decalol-9 was first described by Hückel and Blohm[3] who isolated it from a mixture of both isomers and who indicated its melting point to be 54°. The purest samples of trans-decalol-9 obtained by us by reducing decaline trans-hydro-per-

* Crystallographical and optical analysis of decaline peroxide has been carried out at the Academy of Sciences of the USSR, Institute of General and Inorganic Chemistry, by E. E. Burovaya and G. B. Boky, to whom the author's thanks are due.

** The determination of active oxygen was carried out according to the Hock and Shrader method, somewhat modified by the authors.

oxide had a m. p. of 54.7°, this being explained probably by our samples being more homogenous as they contained no admixture of the cis-isomer, the separation of which, owing to the volatility and solubility of both stereoisomers being almost equal, is extremely difficult.

The Chromic Ester of Trans-Decalol-9 (trans-decalol-9 chromate). A solution of 1 g of decalol (prepared by reducing decaline peroxide with HI) in petroleum ether was shaken for 4 hrs with a solution of 1 g of chromium trioxide in 3 cm^3 of water.

The upper layer, coloured dark orange, was separated, washed with a solution of sodium carbonate, water, dried over ignited sodium sulphate and distilled under vacuum. As the petroleum ether distilled off, needle-like orange crystals fell out. After removing traces of the solvent under vacuum, these crystals melted at 8.68° and decomposed (the trans-decalol-9 chromate, described by Hückel and Blohm, was not quite free from traces of the liquid ester of the isomeric cis-decalol-9, due to the method of preparation used, and melted and decomposed at 78—85° ([2])).

The chromate was reconverted into the initial decalol: 0.6 g of the ester, dissolved in a mixture of 8 cm^3 of glacial acetic acid and 12 cm^3 of water were refluxed for 0.5 hour with 0.5 g of zinc dust; the reaction product was distilled with steam. The crystals that separated after neutralization of the distillate with sodium carbonate were washed with water, dried in a dessicator and sublimed. The resulting substance had a form peculiar to trans-decalol-9—thin leaves with a sharp mint odour—and a melting point of 54.5°. A sample mixed with the initial trans-decalol-9 (m. p. 54.0) had a melting point of 54.4°.

E. G. Mikhailova has taken part in the experimental part of this work.

The Dzerjinsky Heat Engineering Institute.

Received
21.VIII.1944.

REFERENCES

[1] M. Hartmann u. M. Seibert, Helv. Chim. Acta, **15**, 6, 1390 (1932). [2] W. Hückel, Ber., **58**, 1452 (1925). [3] W. Hückel u. M. Blohm, Ann., **502**, 133 (1933).

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CHEMISTRY

PRODUCTS OF LINEAR CONDENSATION OF DIAMINES WITH DICARBOXYLIC ACIDS

By V. V. KORSHAK and S. R. RAFIKOV

(Communicated by A. N. Nesmeyanov, Member of the Academy, 1.IX.1944)

The first systematic study of the polycondensation of diamines with dicarboxylic acids and of the resulting products was made in a number of brilliant investigations by Carothers (1). It is a well known fact that as a result of this work a whole industry of «nylon» products has been created in the USA on the basis of polyamide resin, including high quality fibres, synthetic plastics, *etc.*

Later this subject and particularly the condensation products of hexamethylenediamine with adipic acid were brought up by Staudinger (2) and Sakurada and Hisawa (3).

In our investigation, on the initiative of the late P. P. Shoryguin, Member of the Academy, this reaction and its products have been studied for a number of diamines and dicarboxylic acids (Table 1).

Table 1

Products of Polycondensation of Some Amines and Dicarboxylic Acids

Name of diamine	Name of dicarboxylic acid	Properties of the product		Remarks
		m. p. °C	viscosity 0.02 base mol. solution	
Ethylenediamine	adipic	308—310	low viscosity	The reaction was brought about by heating for 5—6 hrs at a temperature of 225—240° in cresol
»	sebacic	272—274	0.21	
Tetramethylenediamine	adipic	278—280	0.30	
»	sebacic	232—233	0.41	
Hexamethylenediamine	adipic	248—250	0.60	
»	sebacic	207—208	0.87	
Decamethylenediamine	»	194—195	0.58	
Benzidine	adipic	395—400	insoluble	
»	sebacic	375—378	»	

As may be seen from this table, those polyamides which were prepared from hexamethylenediamine with adipic and sebacic acid had the highest molecular weight.

We accordingly paid special attention to the study of the polycondensation reaction itself and of the products obtained from hexamethylenediamine and adipic acid.

The polycondensation of these substances is an equilibrium process, taking place according to the equation

$$\frac{x}{2}\,H_2N(CH_2)_6NH_2 + \frac{x}{2}\,HOOC(CH_2)_4COOH \rightleftarrows$$
$$\rightleftarrows HOOC(CH_2)_4CO[NH(CH_2)_6NHCO(CH_2)_4CO]_{x-2}NH(CH_2)_6NH_2 +$$
$$+ (x-1)\,H_2O$$

Therefore in order to bring this reaction to a high degree, it is necessary to eliminate as completely as possible all the water formed, for instance, by passing a rapid current of nitrogen (free from oxygen) through the reacting mixture. The reaction proceeds better in a solvent (cresol, phenol or xylenol) although it can proceed without it as well. The optimum temperature is within the interval 220—240°

A detailed study of the process has shown that the main factor, determining the degree of polycondensation and the magnitude of the molecular weight of the polyamides formed is the ratio between the initial components, *i. e.* the acid and the diamine.

Flory[4] has shown it possible to calculate statistically the degree of polydispersion, *i. e.* the distribution of the polycondensation products according to their molecular weights, both in the case of equimolecular component ratio and in the case of one of the components being in excess. Flory's calculations for the latter case are very complicated and very difficult to check experimentally.

We confined ourselves to an approximate calculation of the probable degree of polycondensation for the case where one of the components, for instance, adipic acid, was in excess. If it be assumed that the degree of completion of the reaction p is at maximum, *i. e.* the equimolecular amounts of the amino- and carboxy groups have reacted completely, the average degree of polycondensation x of the polyamide formed will depend on the amount of the component taken in excess. If we denote the amount of the component in excess in molar per cent by q, we get

$$q = \frac{100}{x-1}$$

whence

$$x = \frac{100}{q} + 1$$

The calculated values of molecular weights and chain lengths of the polycondensation products of hexamethylenediamine with adipic acid for varying excesses of the acid are given in Table 2.

As this table shows, the reaction is very sensitive to an excess of acid, especially when q is low. This probably explains the discrepancies in individual experiments. In case of large or medium excesses of acid, it may be expected that the products formed will have prevailing chains of one definite length according to the quantity of excess acid or diamine.

A preliminary experimental verification showed that there existed a relation between the acid excess and the molecular weight of the polyamides obtained. We observed that condensation of hexamethylenediamine adipate, when adipic or stearic acid are added, results in the formation of polyamides, the molecular weight of which is in inverse proportion to the excess of the free acid.

It has been found further that the influence of an excess of free acid is manifested not only in the condensation process. If the ready polyamide is heated and free

Table 2

Excess of adipic acid in molar per cent, referred to the sum of moles of acid and diamine, q	Degree of polycondensation (number of segments in the chain), x	Number of carbon and nitrogen atoms in the chain, n	Molecular weight of the chain
50	3	20	372
25	5	34	598
10	11	76	1276
5	21	146	2406
1	101	706	11446
0.5	201	1406	22746
0.1	1001	7006	137206
0.01	10001	70006	1154206
0.001	100001	700006	11324206
0.0001	1000001	7000006	113024206

acid added, the chains are ruptured, and a polyamide of lower molecular weight is formed. This reaction, which we have named «acidolysis», results in a rupture of the peptide bond and probably proceeds according to the following diagram

$$HOOC(CH_2)_4CO[NH(CH_2)_6NHCO(CH_2)_4CO]_x - OH + HOOC(CH_2)_4 - - COOH \rightarrow HOOC(CH_2)_4CO[NH(CH_2)_6NHCO(CH_2)_4CO]_{yy}OH + + HOOC(CH_2)_4CO[NH(CH_2)_6NHCO(CH_2)_4CO]_zOH$$

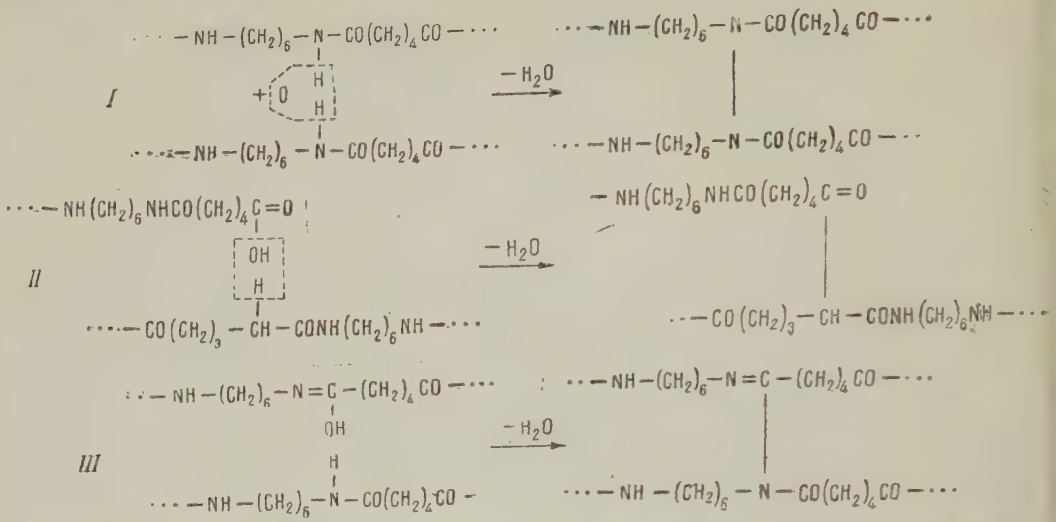
It is interesting to note that the polyamides produced when an excess of acid is added, as well as those formed as a result of «acidolysis», are no longer capable of condensation and are «stable», *i. e.* their molecular weight does not change after being heated in a molten state. In this respect they differ considerably from polyamides produced without an excess of acid; the molecular weight of the latter increases under such conditions owing to a further reaction (involving the end amino- and carboxy-groups).

Thus, the problem of producing polyamides of a required degree of polycondensation can be solved by adding the necessary amount of the carboxylic acid to the initial salt. Thus, for the first time, we have demonstrated clearly the difference in principle between the polycondensation method for preparing compounds of high molecular weight and the second method based on the reaction of polymerization. The first reaction can be regulated, and the results of the process can be theoretically foreseen. The second method is still inaccessible to theoretical foresight, and the result of the polymerization reaction depends on a number of causes which cannot yet be taken into account.

The condensation products of hexamethylenediamine with adipic acid are solid substances, m. p. 248—250°, sp. weight 1.1, insoluble in ordinary solvents. Good solvents for them are phenol, cresols, xylenols and other phenols, as well as formamide and formic acid. They are also soluble in concentrated sulphuric, hydrochloric and nitric acids, in which they gradually hydrolyse. Polyamides are quite inert against the action of alkalies.

Polyamides can be drawn into threads and can form films. This property is peculiar to products whose 0.5 per cent solutions in *m*-cresol have a specific viscosity of about 0.3 and higher. When heated for a long time above their melting point, especially in the presence of atmospheric oxygen, polyamides are easily converted into three-dimensional compounds, insoluble and non-melting.

This transformation may be due to the formation of cross bridges between separate polyamide chains, the result being a three-dimensional network structure. The formation of the latter may be the result of one of the following reactions:



Such three-dimensional polyamides in swelling can bind 100 times its own volume of the solvent.

The molecular weights of different samples of polyamides produced by the authors depended upon the conditions of the experiments and varied from 5 000 to 28 000.

The molecular weights of these samples were determined from their viscosity and from the osmotic effect of their solutions, as well as by the diffusion method. Anomalies were discovered in the behaviour of polyamide solutions in various solvents. These anomalies can be traced to the presence of free amino-groups at the end of the polyamide chains and are due to the capacity of the latter to associate with each other, forming hydrogen bonds, and also to the formation of salts with the carboxyl groups of other polyamide molecules.

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1.IX.1944.

REFERENCES

[1] W. H. Carothers, J. Am. Chem. Soc., **51**, 2548 (1929); W. H. Carothers and J. A. Arvin, ibid., **51**, 2650 (1929); W. H. Carothers and F. J. van-Natta, ibid., **52**, 314 (1930); W. H. Carothers and G. J. Berchet, ibid., **52**, 5289 (1930); W. H. Carothers and G. L. Dorough, ibid., **54**, 761 (1932); W. H. Carothers and J. M. Hill, ibid., **54**, 1579 (1932). [2] H. Staudinger, Melliand Textilberichte, **20**, 631 (1939); H. Staudinger u. H. Jörder, J. f. prakt. Chem., **160**, 176 (1942). [3] J. Sakurada and Hisawa, J. Soc. Chem. Ind. of Japan, **43**, 4386 (1940). [4] P. J. Flory, J. Amer. Chem. Soc., **58**, 1877 (1936).

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PHYSICAL CHEMISTRY

THE NUMERAL SYMBOL OF CLOSE PACKING OF SPHERES AND ITS APPLICATION IN THE THEORY OF CLOSE PACKINGS

By G. S. ZHDANOV

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I. Determination of the Numeral Symbol of Close Packing. A consistent theory of close packings of spheres has been developed by Belov (1), who derived 8 possible space groups for them. This theory is based on the description of close packings by means of alphabetic symbols. A set of 3 letters, A, B, C, indicates the respective distribution of the spheres on 3 different families of axes of symmetry, parallel to the principal axis of packing. That important characteristic of the packing which consists in the fact that its adjacent layers of the spheres can be arranged in a dual manner is expressed by a set of two letters, k and h, (2), cubic and hexagonal, which indicate the dual symmetry of the spheres in the packing.

A close packing of spheres is determined uniquely by the thickness value of the successive layers with parallel and anti-parallel packing

$$\cdots p_1 \cdot p_2 \cdot p_3 \cdots p_j \cdots p_n$$

where P_j is equal to the number of elementary layers of spheres in a layer j with identical types of packing; p_j are positive integers ($p_j \neq 0$). In what follows we consider packings with a finite period. Packings with a periodically recurring succession of layer thicknesses and recurring layer orientation are characterized by the shortest set of numbers, which recurs infinitely in the structure

$$\cdots p_1 \cdot p_2 \cdot p_3 \cdots p_m \cdot p_1 \cdot p_2 \cdot p_3 \cdots p_m \cdots$$

The minimum periodic set of numbers with an even number of terms in the period

$$p_1 \cdot p_2 \cdots p_m \qquad (p_j = 1, 2, 3, \ldots;\ m = 2, 4, 6\ldots)$$

forms the numeral symbol of the given close packing of spheres.

The identity and difference in the numeral symbols is determined by the following rules:

1. Reversal of the order of figures on the symbol, that is writing it the other way round, does not alter the type of packing.

2. Numeral symbols with different combination or alternation of figures represent different packings.

II. Description of Close Packing of Spheres by Numeral Symbol. 1. The number of spheres in an elementary cell and the periodicity of the packing along the shortest translation is $n = \sum_{j=1}^{m} p_j$. The periodicity

along the normal to the plane of the base is $n_c = n$ for hexagonal packings, and $n_c = 3n$ for rhombohedral packings.

2. The ratio of the number of «hexagonal spheres» to the number of «cubic spheres» in the packing is

$$\frac{N_h}{N_k} = \frac{m}{n-m} = \frac{m}{\sum p_j - m}$$

3. The Bravais lattice of the packing is determined by the total difference in the thickness of layers of different orientation

$$\Delta = \sum p_{2k+1} - \sum p_{2k} = n - 2\sum p_{2k}$$

If $\Delta = 3k$, the lattice is primitive hexagonal.

If $\Delta = 3k \pm 1$, the lattice is primitive rhombohedral.

4. The dimensions of an elementary cell of the packing in the hexagonal axes are:

$$c = n_c h, \quad a = \sqrt{3/2}\, h, \quad c/a = n_c \sqrt{2/3} = 0.817\, n_c$$

where h is the height of a tetrahedron formed by four closely packed spheres.

5. The coordinates of the centres of the spheres are expressed as

$$00\frac{p}{n_c}; \quad \frac{2}{3}\,\frac{1}{3}\,\frac{q}{n_c}; \quad \frac{1}{3}\,\frac{2}{3}\,\frac{r}{n_c}$$

for hexagonal packings and as

$$\left(000; \ \frac{1}{3}\,\frac{2}{3}\,\frac{1}{3}; \ \frac{2}{3}\,\frac{1}{3}\,\frac{2}{3}\right) + 00\frac{p}{n_c}; \quad 00\frac{2}{3} + \frac{q}{n_c}; \quad 00\frac{1}{3} + \frac{r}{n_c}$$

for rhombohedral packings.

In both cases p, q, r form a sequence of integers 1, 2, ..., n. The arrangement of these numbers into groups is determined by the following rule. Upon transition from one elementary layer of spheres to another in a layer having the same orientation, the p, q, r replace one another in cycle. Upon transition, in accordance with the packing formula, to a layer having another orientation, the cyclic permutation is reversed.

6. The symmetry elements of the packing are determined by the symmetry of the numeral symbol.

a) If the packing symbol is divided into two equal half-periods (with an odd number of terms in each), then the packing has a two-pitched hexagonal screw axis 6_3 (space group D^4_{6h} and C^4_{6v}). In the rest of the cases there are only axes of symmetry of the third order.

b) If the packing symbol is divided into two symmetrical parts, one the mirror image of the other, with respect to the boundary between layers, then a plane of symmetry passes between these layers, perpendicular to the principal axis (space groups D^4_{6h}, F^1_{3h}).

c) If the packing symbol is so divided, with respect to any layer (figure), then the centres of symmetry will lie in the middle of the layer (space groups D^4_{6h}, D^3_{3d}, D^5_{d3}).

A knowledge of these elements of symmetry is sufficient to determine to which of the seven possible space groups (D^4_{6h}, D^3_{3d}, D^5_{3d}, C^4_{6v}, D'_{3h}, C'_{3v}, C^5_{3v}) derived by Belov the packings belong. Each of these space groups contains an infinite number of close packings of spheres. The eighth possible special group O^5_h —Fm3m contains only one close cubic packing, which is a special limiting case and may be represented by the following formula $(\infty \cdot 0)$.

Classification of Close Packings of Spheres

Δ	$3k$						$3k-1$		S_n	$\sum_{n=2}^{n} S_n$	$N_h : N_e$
Symmetry	Hexagonal			Trigonal			Rhombohedral				
Space group	D_{6h}^4—C 6/mmc		C_{6v}^4—C 6 mc	D_{3d}^3—C $\bar{3}$m	D_{3h}^1—C $\bar{6}$m	C_{3v}^1—C 3m	D_{3d}^5—R $\bar{3}$m	C_{3v}^5—R 3m			
Symmetry elements	×	×		× × ×			× × ×				
n — periodicity 2	1	1							1	1	1 0
3							1 · 2		1	2	1 0.5
4	2	2					3 · 1		2	4	1 1.0
5				1 · 4			2 · 3				1 1.5
							1 · 2 · 1 · 1		3	7	1 0.25
6	3	3					1 · 5 4 · 2		5	12	1 2
					1·2 · 2·1		3 · 1 · 1 · 1				1 0.5
7				2 · 5			6 · 1 3 · 4		8	20	1 2.5
				4 · 1 · 1 · 1 3 · 1 · 2 · 1			1 · 2 · 2 · 2 1·1·1·1·1·2	1 · 2 · 1 · 3			1 0.75 1 0.17
8	4	4		1 · 7			5 · 3 2 · 6		14	34	1 3
				1 · 2 · 3 · 2	1·3 · 3·1		1·1·1·5 1·2·1·4	1 · 1 · 4 · 2 1 · 2 · 2 · 3			1 1
	1·2·1·1·2·1				1·1·2·2·1·1		2·1·1·1·2·1 3·1·1·1·1·1				1 0.33

III. Classification of Close Packings of Spheres. In Table 1 there is given a classification of close packings of spheres built up of equal-sized spheres up to 8 layers in the period inclusive. It does not include the single cubic packing. A characteristic of the structure with respect to periodicity n in the shortest period has been made the basis of this classification. Hence the periodicity of the rhombohedral packing is calculated along the edge of the rhombohedron, not along its body diagonal.

In Table 1 the packings having identical periodicity are divided into subgroups in the order of increasing number of terms m in the symbol. S_n stands for the number of packings with periodicity n.

$\sum_{n=2}^{n} S_n$ gives the number of all packings up to the given periodicity inclusive.

N_h: N_k is the ratio of the number of «hexagonal spheres» to the number of «cubic spheres» in the packing. The elements of symmetry are denoted by the following arbitrary symbols: $\times$, $\times$, centres of inversion; $|'$, $\|$, planes of symmetry.

Packings with a symmetry C_{3v}^1 are first encountered among 9-layered packings and have the symbols 1·1·2·5; 1·2·2·4; 1·1·1·2·1·3. Packings with symmetry C_{6v}^4 appear beginning with $n=12$, namely, 1·2·3·1·2·3.

Most of the types of packings up to 7-layered ones are found in the crystal structures of certain compounds. Types of packings that have been established for binary compounds are shown in Table 2. The most complex of these with a many terms symbol and superperiodicity ($n = 17$), is evidently the one for SiCV [3].

Table 2

Types of Close Packings of Some Binary Compounds

n	1	2	3	4	4	5	6	17
Packing symbol	(∞ · 0)	1·1	1·2	1·3	2·2	2·3	3·3	2·3·3·3·3·3
Compound	sphalerite	würzite	Al_4C	$CaSi_2$	SiCIII	SiCI	SiCII	SiCV

The elements preferring a close packing, such as metals and the inert gases, crystallize in the simplest types: cubic (∞.0) and hexagonal (1·1).

Conclusions. For a characteristic of close packing of spheres a numeral symbol has been proposed that fully describes the geometrical properties of the packing (periodicity, Bravais lattice, cell dimensions, atom coordinates, space group). The numeral symbol simplifies classification of close packings and facilitates the interpretation of closely packed complex structures.

I take this opportunity to express my gratitude to Dr. N. V. Belov for going over and discussing this paper before its publication.

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REFERENCES

[1] Н. В. Белов, Диссертация, ИОНХ АН СССР, 1943. [2] N. V. Belov, C. R. Acad. Sci. URSS, XXIII, 170 (1939). [3] G. S. Zhdanov and Z. V. Minervina, ibid., XLVIII, No. 2 (1945).

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GEOCHEMISTRY

AN ATTEMPT TO CORRELATE THE OILS AND BITUMENS OF THE URAL-VOLGA REGION ACCORDING TO MICROELEMENTS

By L. GULIAEVA

(Communicated by S. S. Nametkin, Member of the Academy, 19.X.1944)

In the course of a study of the composition of the ash of the oils from the Ural-Volga region the author has observed and reported in a previous communication (1) a high content of vanadium, nickel and copper in these oils and a definite regularity in the proportion of elements: V > Ni > Cu.

In continuing her investigation, the author made an attempt to determine these proportions with higher precision and to find out whether and to what extent the regularities revealed hold true for a definite group of oils and natural bitumens. In addition to the previous analyses the author made use of the results of an investigation of the ash of oils and bitumens of the Permian deposits of the Tartar ASSR, as well as of those from some other regions, carried out in the laboratory of geochemistry and hydrochemistry of oil deposits at the Institute of Mineral Fuel of the Academy of Sciences of the USSR.

The author was able to compare the ratios of vanadium, nickel and copper in the oils and certain solid bitumens for a vast area of the Ural-Volga region in different stratigraphical horizons, from the Lower Carboniferous to the upper portion of the Permian deposits. This correlation of the material available has led the author to a conclusion that the ratios of nickel and copper in oils and bitumens are highly variable; though nickel always predominates over copper, the Ni to Cu ratio varies within a wide range from 3 to 500, these variations having no relation to the stratigraphic horizon of oil occurrence. The V to Ni ratio was found to be a rather constant value, characteristic of whole groups of oils and bitumens.

The vanadium and nickel content in the oils and their ratios are given in Table 1. The data on the vanadium content have been taken partly from a previous paper by this author (1), partly from a paper by Romm (2).

An examination of the data given in Table 1 will show that according to the value of the V to Ni ratio the oils investigated fall into two large groups, namely:

1) Oils with a V to Ni ratio below 3, or but sligthly exceeding 3; these are the oils of the Carboniferous and Lower Permian of the Artinskian and Kungurian deposits. The average value of the V/Ni ratio in these oils is 2.5.

2) Oils with the V/Ni ratio above 3, ranging between 4 and 7. These are the oils of the Upper Permian deposits. The average value of the V to Ni ratio in these oils is about 5.

The value of the V to Ni ratio in the two above groups of oils is constant enough. Only the light oils of Severokamsk and Polazna (specimens Nos. 7 and 8, Table 1) are distinguished within the 1st group of oils by a markedly lower content of vanadium and nickel, and a decrease of the V to Ni ratio below 1.

Table 1

V to Ni Ratio in Oils

Nos.	Place of sampling	Horizon	Specific gravity	V, mg per 100 g oil	Ni, mg per 100 g oil	V/Ni
1	Tuimazy, well 30	C_1^{tur}	0.8998	8.39	3.7	2.27
2	» well 36	C_1^{h}	0.9100	12.7	4.3	2.95
3	Syzran, well 27	»	0.900	7.15	2.59	2.76
4	Yablonovy Ravine, well 1	»	0.8890	5.9	1.85	3.18
5	Krasnokamsk, well 1	»	0.8620	1.18	0.71	1.66
6	» well 36/5	C_2^1 (martjan)	—	1.94	0,75	2.58
7	Severokamsk, well 3	C''	0.8500	0.87	0.67	1.29
8	Polazna, well 1	»	0.8400	0.06	0.25	0.24
9	Syzran, well 15	C_2^1	—	5.6	1.85	3.02
10	Krasnokamsk, well 13	P_1^{kg}	0.9500	4.16	1.58	2.62
11	Ishimbai (oil mixed), from a tank	P_1^{rt}	—	4.43	1.49	2.95
12	Ishimbay, well 6/2	»	—	3.18	1.83	1.77
13	» , well 729/19	»	—	4.69	2.0	2.34
14	Zmievo, well No. 3 bis	P_2^{uf}	<0.937	61.0	10.0	6.1
15	Sarabikulovo, well 5	»	—	19.8	4.8	4.1
16	» well 1 bis	$P_2^{kar_1}$	—	15.87	2.6	6.06
17	Buguruslan, well 23	»	0.9000	10.8	2.33	4.63
18	Novo-Stepanovka, well 150	»	0.8790	6.59	1.27	5.78
19	» well 360	»	0.8800	7.81	1.07	7.30

The author has analysed also some specimens of asphaltites from the deposit near the village Sadki and from the Syzranka River (the Samara Bend). The former asphaltite occurs as a thick vein filling a very large fissure in Tartar deposits; the latter occurs in the dolomites of the Upper Carboniferous along the Syzranka River near the town of Syzran, filling fissures and caverns and as bands along the bedding planes. The results of the analyses are given in Table 2.

Table 2

Name of specimen	Horizon of occurrence	Ash, %	V, mg per 100 g asphaltite	Ni, mg per 100 g asphaltite	V/Ni
Asphaltite, Sadki village, specimen No. 2	P_2^{tat}	1.36	250.0	39.0	6.41
Do, specimen No. 1	»	1.08	341.0	65.8	5.19
Apshaltite, from the Syzranka River, specimen No. 1	C_3	1.61	72.0	18.9	3.81
Do, specimen No. 2	»	4.2	52.4	17.2	3.04
Do, specimen No. 3	»	1.69	54.0	21.0	2.57

An examination of Table 2 will show that both asphaltites are characterized by a high content of vanadium and nickel, particularly the Sadki asphaltite, in which the vanadium content reaches 341, and that of nickel, 65 mg per 100 g of asphaltite. With such a high content, the V/Ni ratio in the Sadki asphaltites is equal to the value of that of the Upper Permian oils (Novo-Stepanovka, Table 1, specimens Nos. 18—19), and the V/Ni ratio in the Syzran asphaltite approaches that in the oils of the Carboniferous (Syzran, Table 1, specimens Nos. 3 and 9).

The constancy of the V/Ni ratio in the Upper Permian oils and bitumens is of particular interest. Among these caustobiolites there are both liquid, mobile oils (Buguruslan, Novo-Stepanovka), and heavy oils, containing scarcelly any low-boiling fractions (Zmievo) and, finally, solid asphaltites. Aside from the difference in their physical properties, they are also characterized by different modes of occurrence. The oils of Buguruslan and Novo-Stepanovka occur in Lower Kazan

deposits at a depth of about 270 m, are very well isolated from the day surface by an impermeable cover composed of thick anhydrites, and are surrounded by waters very rich in hydrogen sulphide. The Zmievo oil occurs in sandstones amidst red Ufa clays, at a depth of about 87 m; lastly, asphaltite from the Sadki village directly outcrops to the surface. However diverse geological setting may be, all these caustobiolites have a feature in common, namely, the value of the V/Ni ratio. This feature is, of course, no casual one and characterizes the specific chemical structure of the organic matter of Upper Parmian bitumens, owing to which vanadium and nickel enter its composition in definite proportions. The V/Ni ratio is thus a genetic characteristic which makes it possible genetically to correlate different caustobiolites according to the value of this ratio.

Using the value of the V/Ni ratio as a character for such correlations, it is necessary to know what are the natural processes by which it can be affected. Such processes are the adsorption during the migration of oil in the rock and high oxidation proceeding under participation of water and oxygen.

In the processes of adsorption which take place, for instance, when the oil is making its way through a bed, it is asphaltenes and tars which are adsorbed by the rock in the first place, and along with these vanadium and nickel confined, as is known, to these fractions of oil. But as nickel is associated with a wider fraction, while vanadium is restricted to the most high-molecular portion of oil ([3–5]), vanadium will be removed from oil at a more rapid rate than would nickel, owing to the selective adsorption of the heaviest fractions of oil by the rock. As a result, the oil will become poorer both in vanadium and in nickel, but chiefly in vanadium, and the V/Ni ratio will be shifted towards a decrease. A process of this kind may account for the decrease in the V/Ni ratio in the oils of Severokamsk and Polazna, if it be assumed that there a migration of the oil through the bed had taken place in a direction from Krasnokamsk to Polazna.

The process of a deep oxidation of bitumen in which water and oxygen take part, brings about a removal of the light fractions from the bitumen and the formation of humus-like substances ([6]), which is characteristic of the burnt ones.

S. P. Uspensky has experimentally established that both vanadium and nickel, but mainly nickel, are washed out from the bitumen, even to its complete removal. In this way this process, as opposed to the adsorption process, brings about a shift of the V/Ni ratio towards an increase.

So far advanced processes of oxidation seem to be absent in oils, but they may take place in solid bitumens. It may be stated that in solid bitumens investigated by the author no processes of such deep oxidation have occurred, since nickel is present in them in considerable amounts. In the Sadki asphaltite no shift is observed in the value of the V/Ni ratio as compared to the Upper Permian oils.

Thus, the value of the V/Ni ratio in oils and bitumens of the Ural-Volga area permits:

1) to distinguish among these caustobiolites two genetic groups;

2) to establish the absence of far advanced processes of oxidation in the asphaltites of Sadki and the Syzran River and the genetic distinctions between these caustobiolites;

3) to obtain evidence of a migration of oil through the bed in the region of the Near-Kama deposits of the Middle Carboniferous.

In conclusion it should be noted that the investigation of the composition of the ash of oils as a whole, not restricted to a narrow group of elements, is of great interest and may prove very useful for the study of the origin of natural caustobiolites.

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REFERENCES

[1] L. A. Guliaeva, E. S. Itkina, I. I. Romm, C. R. Acad. Sci. URSS, XXXII, No. 6. (1941). [2] И. Ромм, Диссертация, Фонды ИГИ АН СССР, 1944. [3] W. Ramsay, J. Inst. Petr. Technol., **10**, No. 41 (1924). [4] W. B. Shiery, Ind. Eng. Chem., **23**, No. 10 (1931). [5] А. П. Виноградов, Сборн. «Академику Вернадскому», **1**, 1935. [6] Н. А. Орлов и В. А. Успенский, Минералогия каустобиолитов, 1936.

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GEOLOGY

GENETIC TYPES OF CRYSTAL-BEARING «CELLARS» AND QUARTZ VEINS OF ALDAN

By N. P. ERMAKOV

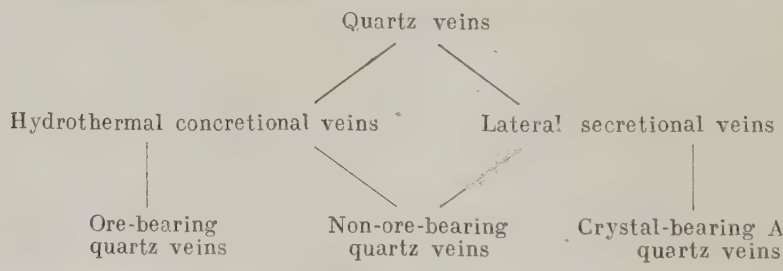
(Communicated by A. E. Fersman, Member of the Academy, 30.IX.1944)

The genesis of rock-crystal deposits has until now been very little discussed in literature.

Still less attention has been paid to making out the types of crystal-bearing veins and «cellars». The present brief note, based chiefly on a study of the Aldan deposits, is concerned with these questions, which are of essential importance for the economic evaluation of deposits of rock crystal.

Among the papers of Russian writers, some particular aspects of the crystal genesis of quartz are dealt with in the interesting papers by Lemmlein ([5,6]) and Vertushkow ([1]), while the general problem as to the origin of the rock-crystal deposits of the Near-Polar Urals is discussed in a paper by Merkulova([9]). Among the papers of foreign writers, most important data on the genesis of the deposits in question are contained in the works by Koenigsberger ([3,4]).

The principal genetic types of quartz veins may be presented by the following scheme:



According to the origin of the mineral components among the quartz veins two chief groups are distinguished: hydrothermal-concretional veins formed by the deposition of magmatic distillations, and lateral-secretional veins formed as a result of hydrochemical processes of solution and redeposition of the material of the enclosing rocks by the hot waters of various origin circulating within these rocks.

The characteristic features of the ore-bearing, non-ore-bearing and rock crystal-bearing types of quartz veins are given below (Table 1).

From the standpoint of the commercial value of rock crystal the ore-bearing veins are of no importance whatever. The non-ore-bearing quartz veins are extensively developed and sometimes play a large part in the formation of piezo-quartz deposits in different regions.

The latteral-secretional rock crystal-bearing veins form the most valuable piezo-quartz deposits.

Table 1

Characters	Ore-bearing quartz veins	Non ore-bearing quartz veins	Crystal-bearing quartz veins
Ore minerals	Often contained at commercial concentrations	Usually absent or occurring as a scarce impregnation	Absent or rarely present in rock-crystal individuals or in the quartz fringe
Crystal-bearing voids	Usually absent or rarely present as very small voids with small rock-crystal individuals	Inside the vein quartz often occur voids with rock-crystal individuals at commercial concentrations	As a rule, include considerable cavities with rock-crystal individuals
Vein filling	Greyish-white or coloured, massive or spongy quartz, sometimes with a variable structure	Milk-white, sometimes vitreous, usually even-grained, dense, less frequently with a comb- or columnar structure. In the quartz sometimes occur vein carbonates, sericite or chlorite	Quartz does not form a continuous vein body, but only fringes on walls of fissures and cellars filled with crystals and frequently with clay
Contacts with the enclosing rocks	Contacts usually sharp, distinct, but in the case at metasomatosis phenomena the edges of the veins become diffuse and sinuous, and the enclosing rocks, altered	Nature of contacts depending on enclosing rocks	Contacts are uneven, sinuous. Quartz fringes generally grade into enclosing rocks, sligthly ultered at «salbands»

In the Alps, Koenigsberger separated out a special formation of veins of the Alpine type ([3,4]) on the basis of their peculiar geological environment, paragenesis of minerals and the established constancy in the sequence of their formation, recurring regularly for a large number of veins. The rock-crystal lateral-secretional veins developed in the Aldan region are essentially Alpine veins; at the same time then have a number of points of distinction, worthy to be mentioned, which are given in Table 2.

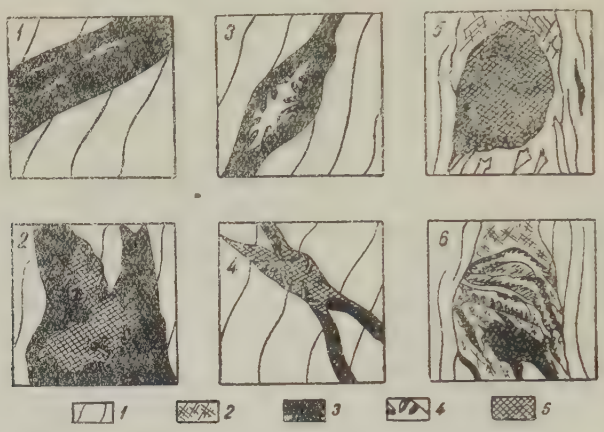
Table 2

Characters	Veins of Alpine type	Aldan crystal-bearing veins
Enclosing rocks	Highly variable, containing from 45 to 70% silica	Uniform quartzite, containing over 95% silica
Mineral composition	Highly variable. Up to 60 mineral species found	Uniform. Up to ten mineral species found
Relation to structures	Relation of veins to gentle tension joints, cutting the structures	Relation of veins to fractures and zones of schistosity superimposed upon structures, generally having a dip approaching the vertical
Morphology and size of veins	Lens-shaped, sometimes lenticular veins of a small size up to 20—30 m long are characteristic	Vein zones and veins extending over several hundreds metres or short vein bodies of indefinite shape are characteristic

A certain peculiarity of the Aldan rock-crystal veins follows from a difference in the manifestations of fracture tectonics and from an exceptionally high silica content in highly uniform enclosing rocks. This fact has resulted in an absolute

predominance of rock crystal among the vein minerals, for which reason the Aldan veins have become deposits highly favourable for the formation of piezo-quartz.

The ore-bearing gold carrying quartz veins in the Aldan area, according to the views of prominent American geologists, have been formed by deposition of silica from colloidal solutions of a primarily magmatic origin ([7], [8], [10]). The rock crystal-bearing veins of the Aldan area have been formed almost exclusively at the expense of the solution of the enclosing rocks with hot waters of various origin and a redeposition of silica from true solutions. It seems that a large part in the formation of these veins was played by «metamorphogenesis» ([2]) waters separated out in the process of a metamorphism of the enclosing rocks. Genetically the non-ore-bearing



Types of crystal-bearing-pockets and «cellars» (cross-section through quartz veins). *1*, enclosing rocks; *2*, schistosity zones; *3*, quartz veins; *4*, rock-crystal individuals; *5*, preserving clay

quartz veins occupy an intermediate position between the ore-bearing and crystal-bearing veins, and are formed at the initial stage, as it seems, by a deposition of silica gel in fractures and subsequently of silicon dioxide in the voids of the vein in the process of its formation; the silicon dioxide is precipitated from true solutions and formed crystals with well-developed faces of a perfect structure. The diagenesis of the vein matter deposited at the first stage is of great significance in the formation of cavities inside the vein and in the later formation of rock-crystal pockets, partly derived from residual solutions of a low concentration. The genetic and morphological types of voids, where takes place formation of rock-crystal individuals, are classified according to their genesis and type of rock-crystal veins in Table 3.

Table 3

Genetic types of voids

Intervein cavities	Tectonic fissure voids	Leaching caverns in zones of schistosity of rocks
1. Diagenetic lens-shaped cavities 2. Irregular cavities of incompletely filled fissures	3. Lens-shaped voids of shear fractures 4. Irregular voids at intersections of fractures	5. Irregular tubular caverns of leaching 6. Multi-chamber «cellars» with an arch stoping

While the origin of fissure voids and caverns of leaching is a phenomenon rather easy to understand, it is very difficult to account for the origin of intervein cavities. The formation of the latter from a number of characteristic features should probably be related to a considerable decrease in the volume of the substance occurring upon the syneresis of the silica, filling some vein fissure or other. Most crystal-bearing pockets and cellars are generally filled with a clayey material which is the result of a leaching of the enclosing rocks with later phreatic, metamorphic and, possibly, magmatic fissure solutions.

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REFERENCES

[1] Г. Н. Вертушков, Зап. Мин. об-ва, LXXI, No. 1—2 (1942). [2] A. N. Zavaritsky, Bull. Acad. Sci. URSS, sér. géol., No. 3 (1943). [3] J. Koenigsberger, Abh. Bayer. Acad. Wiss., Math. phys. Klasse, 25 (1917). [4] J. Koenigsberger, Schweiz. Min. Petr. Mitt., VIII, H. 1 (1928). [5] G. G. Laemmlein, C. R. Acad. Sci. URSS, XXII, No. 1 (1939). [6] G. G. Laemmlein, Bull. Acad. Sci. URSS, sér. géol., No. 5 (1927). [7] В. Линдгрен, Сборн. «Рудные магмы», 1933. [8] В. Линдгрен, Минеральные месторождения, вып. 1, 1934. [9] G. V. Merkulova, Bull. Acad. Sci. URSS, sér. géol., No. 3 (1942). [10] Д. Сперр, Сборн. «Рудные магмы», 1933.

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GEOLOGY

THE MURZHIK DEPOSIT OF MANGANESE ORES

By P. P. MIKHAILOV

(Communicated by V. A. Obruchev, Member of the Academy, 30.IX.1944)

The present paper contains the results of revisions and a reconnaissance of the Murzhik deposit of manganese ore, carried out by the author in the summer of 1943.

The Murzhik deposit is located on the south-western spurs of the Murzhik Mts., 270 km from the Zhana-Semey station, Turkestan-Siberian railway, and 150 km from the Maiskoye wharf on the Irtysh River.

The area of the deposit is composed of sedimentary metamorphic and igneous rocks of Palaeozoic age. The oldest deposits of the region are metamorphic schists of the Murzhik series, composing the northern marginal part of the region (Murzhik Mts.).

From stratigraphic comparisons it has provisionally been referred to the Lower Palaeozoic (older than S_2). The schists of the Murzhik series are crumpled in isoclinal folds of a north-western strike, and broken by intrusions of igneous rocks of a medium and basic composition. The contact of these old rocks with younger deposits is a tectonic one.

The most extensively developed in the region are rocks of the Devonian, composing the western, southern and eastern marginal parts of the region: the Devonian deposits consist of diverse effusives, their tuffs and red sandstones. These deposits are separated from the overlying limestones of the Carboniferous (Tournaisian) by a thin series of rocks of the «ore-bearing horizons», whose composition is highly complicated, variable, and characterized by a rapid thinning-out along the strike of the component tuffites, sandstones, gaize-like rocks, conglomerates, bituminous limestones and manganese ores represented by oxide and carbonate varieties. The manganese ores are confined to the rocks of the ore-bearing horizon only, and do not occur in any other horizon of this series of rocks of the region.

The central part of the region is composed of limestones of the Tournaisian, with a total thickness of 600—700 m; they contain a rich fauna (brachiopods, bryozoans).

The section of the series begins with granular greyish-pink limestones, conformably overlying the rocks of the ore-bearing horizon. In the middle horizons of the series they are superseded by light-grey silicified varieties containing bands of finely shelly and white crystalline limestones. The upper horizons of the carbonate series are represented by marlaceous limestones. The latter are conformably overlain by deposits of the coal-bearing series of the Visean, represented by coarse-grained sandstones with numerous remains of flora (*Lepidodendron*, *Stigmaria*, stems of equisetites, *etc.*) and coal seams (Ak-Cheku deposit).

The rocks of the region of the deposit form a rather large brachysynclinal fold, with the axis elongated in a north-western direction. The limbs of the major fold

are complicated by a minor folding and broken by a number of tectonic disturbances of a small amplitude. The northern part of the trough is cut away by a large tectonic disturbance causing a thrusting of the metamorphic schists of Murzhik over the rocks of the Devonian and Carboniferous.

The dislocated Palaeozoic deposits are unconformably overlain by a mantle of piedmont deposits of Tertiary or Old Quaternary age.

The dislocation of Middle Palaeozoic deposits seem to be associated with one of the first phases of Variscian folding. Later phases of the latter were displayed, as it seems, in the form of disjunctive disturbances.

In the process of revisory geological search among the rocks of the ore-bearing horizon, along with the previously known Esym-Zhal area, manganese ore beds were discovered in three other areas.

The most promising area of the Murzhik deposit is Esym-Zhal, located in the north-western part of the Murzhik trough. With the aid of trending the ore bed has been traced on the surface over a distance of 3 km.

The rocks composing the ore-bearing horizon are rather variable and change rapidly along the strike. In the southern and central part of the area the ore bed is underlain by brick-red sandstones and conglomerates, while in the hanging wall of the ore bed there occurs a layer of tuff-sandstones overlain by greyish-pink granular limestones of the Lower Carboniferous. At some places amidst tuff-sandstones there occur layers of white gaize-like rocks, claye shales and bituminous limestones. In the northern half of the area the ore bed stratches for the of 1500 m amidst bituminous limestones underlying coarse-grained tuff-sandstones. The strike of the ore bed in the southern half of the Esym-Zhal area approaches a meridional one, dipping East at angles of 75—90°. In the northern half of the area the ore bed has a north-eastern strike and dips north-westward at angles of 50—75°, that is, has a reversed dip. The thickness of the ore bed varies from 0.4 to 7.5 m in the southern part of the area (on the average, 2.7 m), while in the northern part the bed is thinning out.

The ores of the Esym-Shal area are comparatively high in manganese (up to 30—35 per cent), which is occasionally space dropping to 20 per cent.

On the surface the ore bed is composed of pulverulent sooty rocks, not infrequently containing pieces of a solid rock. The mineralogical composition of the ores has not been sufficiently studied as yet. Psilomelane, wad, pyrolusite, braunite, rhodochrosite, probably manganite have been macroscopically determined. Among the non-ore minerals, gypsum, calcite, quartz, barite and kaolinite are widespread.

In December, 1943, in the central part of the Esym-Shal area a hole was sank with the core drill; it penetrated the ore bed within the range from 23 to 29 m. The drill-hole records indicate a certain increase in the thickness of the bed with depth, a gradual smoothening of the dip and preservation of the mineral composition of the ore (braunite, psilomelane, wad, pyrolusite). Down to a depth of 30 m there is no decrease in the manganese content of the ore.

The ores of the northern part of the Esym-Zhal area are distinguished by their lower grade and lesser persistance. Here the bed of oxide ores has a smaller thickness (about 1.6 m) and frequently thins out completely, being replaced along the strike by carbonate ores.

The latter are distinguished by a very low manganese content of 2.0—6.0 per cent, and seem to be represented by mangano-calcite.

The ore areas of Kos-Taran, Uzun-Bulak and Dauletpai, located in the vicinity of the Esym-Zhal area and confined to the same strigraphic horizon as the latter are distinguished by a smaller thickness of the oxide ore bed, its frequent wedging-out, and a considerable development of carbonate ores.

Until now the above-named areas have not been sufficiently investigated, but according to preliminary data they undoubtedly contain ores less rich than those of the Esym-Zhal area.

The oxide ores of these areas are characterized by a lower manganese content—from 20 to 25 per cent and lower.

The data of a preliminary investigation of the ore areas of the Murzhik deposit enable one, in the author's opinion, to make a conclusion as to the primary sedimentary origin of manganese ores which were subsequently subjected, along with the enclosing rocks, to regional metamorphism.

The manganese ores are quite distinctly confined to a definite stratigraphic horizon located at the boundary between the deposits of the Upper Devonian and the Lower Carboniferous. The facial conditions corresponding to this time were rather diversified and changed rapidly in time and space, owing to wich the lithological composition of the rocks of the ore-bearing horizon is rather variegated. The deposition of the manganese ores was accompanied by breaks, a thinning-out of the ore beds and a frequent substitution of the carbonate rocks by oxide rocks, and *vice versa*.

From the geological and economical standpoint the Murzhik deposit of manganese ores has not been sufficiently studied. However, even the present state of knowledge shows this deposit to be one of considerable interest for the organization of extensive search and prospecting for manganese in the area of the deposit, and for starting a detailed prospecting in the Esym-Zhal area and reconnaissance in the Uzun-Bulak, Kos-Taran and Dauletpai areas. If these investigations furnish favourable results, this deposit will possibly become an abundant source of manganese for the ferrous metallurgy of Kazakhstan and East Siberia.

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MINERALOGY

MITRIDATITES FROM THE KERCH AND TAMAN PENINSULAE

By A. V. SIDORENKO

(Communicated by A. E. Fersman, Member of the Academy, 10. X.1944)

The ore beds of the Kerch and the Taman Peninsula are frequently found to contain various earthy intercalations of dark or light-green shades.

In 1910 Popov (1) described a light-green substance from the Kamysh-Burun mine (see table), which proved to be a calcium ferriphosphate differing in composition from any other calcioferriphosphate known. In 1914 Dvoichenko (2), on the basis of this analysis, gave this mineral the name of mitridatite and calculated a formula which was unfortunately disfigured by a misprint $(CaMnMgFe)O \cdot 2Fe_2O_3 \times \times P_2O_5 \cdot nH_2O$ and became widely known in this form (3,4). As a matter of fact, from the values of Popov the following formula is obtained: $(CaMgMnFe)O \cdot Fe_2O_3 \times \times P_2O_5 \cdot 3H_2O$. Much later the green calcioferriphosphates from Kamysh-Burun were studied by Chukhrov (5) who published analyses of the green substance named by him mitridatite, although it differed in composition from Popov's phosphate in a somewhat higher CaO content. Chukhrov suggests the following formula of mitridatite: $3CaO \cdot 2F_2O_3 \cdot 2P_2O_5 \cdot 5H_2O \cdot n$ aq.

The difference in the formulae made the present author undertake a study of this mineral which is extensively developed in the deposit named above. He collected specimens of green phosphates from the Kerch and the Taman peninsulae; the result of their analyses are given below.

Phosphate from the Kamysh-Burun Mine. In the upper part of the ore bed the ore is enriched in green calcioferriphosphate; at places occur even thin intercalations consisting of crusts and small nodules, often enveloping oolites of brown ironstone. The superficial portions of the crusts are loose, light-green, with a hardness 1. The inner portions are dark-green, dense, with a hardness 2.5. Streak, brown-green; powder, dark-green.

Under the microscope the mineral contains hardly any impurities of iron oxides; calcite grains are rarely encountered. It consists of small irregular-shaped clods. The colour of the fine clods is light-yellow with a greenish shade, and that of the thick ones, brown-yellow, greenish. At a high magnification some evidence of a granular cloddy structure is observed. It is not birefringent, but some detached areas of the substance affect polarized light, which is evidence of the metacolloid state of the substance.

Jointly with mitridatite a brown-black glue-like substance with a resinous lustre is encountered, resembling the substance reported by Chukhrov(5).

The mineral is soluble in HCl, HNO_3 and H_2SO_4 upon heating. Upon ignition it becomes brown-red; it does not melt.

For the analysis reported in the table, only dark-green copmact pieces were carefully selected under the magnifying lens, the light-green substance and limonite being removed.

Constituents	Theoretical composition from formula: $RO \cdot Fe_2O_3 \times P_2O_5 \cdot 6H_2O$	S. P. Popov, 1910 Kamysh-Burun			F. V. Chukhrov, 1937 Kamysh-Burun			A. V. Sidorenko Kamysh-Burun			A. V. Sidorenko Zheleznaya Balka	
		analysis	in pure substance	molecular ratio	analysis (average from 2 determ.)	in pure substance	molecular ratio	analysis	in pure substance	molecular ratio	analysis (average from 2 determin.)	in pure substance
CaO	12.05	12.43	7.58	0.72	12.26	12.04	1.24	11.52	10.69	0.99	10.62	12.70
MgO		traces	traces		0.82	0.82		traces	traces		—	—
MnO		2.10	2.32		0.79*	0.91		0.25	0.27		0.65	0.78
FeO		0.29	0.32		0.78	0.90		0.90	0.97		2.58	3.08
Fe_2O_3	34.27	37.81	42.56	1.12	31.54	36.40	1.14	40.06	43.27	1.28	28.12	33.68
P_2O_5	30.49	30.09	33.86	1.00	24.48	28.45	1.00	27.67	29.88	1.00	19.18	22.98
CO_2	—	5.28	—	—	none	—	—	1.29	—	—	not determined	—
SO_3	—	—	—	—	2.66	—	—	traces	—	—	—	—
H_2O	23.19	11.83	13.32	3.1	—	—	—	13.82	14.92	3.94	22.40	26.78
Loss on ignition	—	—	—	—	18.99	20.48	5.74	—	—	—	—	—
Insoluble residue	—	—	—	—	8.19	—	—	4.92	—	—	15.24	—
Total	100	99.83	99.96	—	100.40	100.00	—	100.43	100.00	—	98.79	100.00

* 0.88% MnO_3 are recalculated to MnO.

The analysis gives the following formula: (CaMnFe) $O \cdot Fe_2O_3 \cdot P_2O_5 \cdot 4H_2O$. As will be seen from the table, containing also the analysis of Popov's phosphate, as well as from personal information of Popov, the two specimens are identical both in chemical composition and external appearance. From the values of Popov the present author calculated a formula closely approaching that suggested by this writer: $(CaMgMnFe)O \cdot Fe_2O_3 \cdot P_2O_5 \cdot 3H_2O$.

Aside from these two analyses similar green phosphate has also been described by Chukhrov ([6]) among the derivates of anapaite. This mineral in all its properties is analogous to that of the present author (see table). Its formula is $(CaMnFe)O \times \times Fe_2O_3 \cdot P_2O_5 \cdot 6H_2O$.

The Phosphate from the Zheleznaya Ravine (Balka). South of Taman, at the Zheleznaya ravine, the author discovered a green loose calcioferriphosphate. The mineral occurs in the upper part of the ore bed as friable earthy mass with slightly cemented small clods readily converted to powder. It is smeary, dark-green. Both macro- and microscopically, the mineral is identical with the phosphate from Kamysh-Burun. The chemical composition is given in the table.

Because of the large insoluble residue of the substance the author refrains from any interpretation of its chemical composition. It will only be noted that it closely resembles any other phosphate of this kind.

When comparing the chemical analyses of different green phosphates from the Kerch and the Taman peninsulae (see table) described under the names of mitridatite (Chukhrov), or green calcium ferriphosphate (Popov), or of the phosphates from the Zheleznaya ravine and Kamysh-Burun mine (Sidorenko), it will be seen that all these substances closely resemble one another and are hardly distinguishable by their external appearance. Their chemical composition is characterized by the same constituents, though in proportions varying within a range admissible for colloids, which are somewhat contaminated, as a rule. Moreover, for the analysis air-dry samples were used, which might possibly contain different amounts of hygroscopic water.

Thus, at present there are two green phosphates sufficiently characterized from the chemical standpoint and somewhat different in composition; for one of them the author has calculated the formula $(Ca,Mg,Mn,Fe)\ O \cdot Fe_2O_3 \cdot P_2O_5 \cdot 3—6H_2O$. The other, mitridatite after Chukhrov, has a formula $3(Ca, Mg, Mn, Fe)\ O \cdot 2Fe_2O_3 \times \times 2P_2O_5 \cdot 5H_5O \cdot n$ aq. As the minerals have been studied only chemically, and no thermal or X-ray investigations have been made, the author does not consider it rational to separate out each variety as an independent mineral species on the basis of analyses only. It would be more rational to unite them in a common group of mitridatites with a formula

$$(Ca,\ Mg,\ Mn,\ Fe)\ O \cdot Fe_2O_3 \cdot P_2O_5 \cdot 3—6H_2O$$

The ratio of metal atoms to phosphorus in mitridatite is 3 : 2; which is characteristic of many phosphates. The $RO : R_2O_3$ ratio is 1 : 2; in anapaite their ratio is 2 : 1.

The mineral is formed in the case of the weathering of anapaite, when the Ca content decreases, while the amount of $Fe^{\cdots}$ increases ([6–8]). It forms pseudomorphs after anapaite.

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REFERENCES

[1] С. П. Попов, Тр. Геол. музея АН СССР, IV, вып. 7 (1910). [2] П. А. Двойченко, Зап. Крым. о-ва естествоисп. и любит. природы, вып. 1 (1914). [3] И. И. Танатар, Определитель минералов под микроскопом, 98, 1935. [4] О. М. Шубникова, Тр. Ломонос. ин-та АН СССР, вып. 7 (1936). [5] Ф. В. Чухров, там же, вып. X, 226 (1937). [6] Ф. В. Чухров, там же, вып. X, 131 (1937). [7] A. V. Sidorenko, Bull. Acad. Sci. URSS, sér. géol., No. 6 (1940). [8] Ф. В. Чухров, Тр. Ломонос. ин-та АН СССР, вып. 7 (1936). [9] Ф. В. Чухров, Коллоиды в земной коре, изд. АН СССР, М., 1936. [10] С. П. Попов, Минералогия Крыма, изд. АН СССР, М., 1938.

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GENETICS

PRODUCTION OF THE ORIGINAL SPECIES OF THE CHILE AUTOTRIPLOID *SOLANUM MAGLIA* SCHLECHTD. AT PAMIR

By R. L. PERLOVA

(Communicated by A. A. Richter, Member of the Academy. 13.IX.1944)

The wild triploid potato, *Solanum maglia* Schlechtd., ($2n=36$), belongs to the group *Eutuberosa* Juz. and occurs in Chile at Valparaiso at an altitude of 500 (m^{1-3}.) The origin of *S. maglia* is unknown. The absence of diploids and occurrence only of tetraploid forms within the group *Eutuberosa* in its area in Chile precludes the possibility of a hybrid origin. Most probably it is an autotriploid which had arised through chromosomal mutation from a diploid ancester which became subsequently extinct or still unknown.

Solanum maglia as a consequence of its triploidy is self-sterile under various conditions—in Leningrad, in Moscow, at Maikop and, probably, in its native land. When grown in Leningrad, at the Exper. Station of the All-Union Institute of Plant Industry (VIR), sometimes it set seed balls from self-pollination but none of them contained no seeds. Seedless balls were obtained 1938 at the North Caucasus at two branches of the Maikop Experiment Station of VIR, namely, at Shestak at an altitude of 500 m, and at Khadjokh at an altitude of 900 m. Workers of VIR in Leningrad succeeded in producing seed balls on rare occasions by pollinating *S. maglia* with the pollen of other species, especially of the tetraploid ones: *S. tuberosum* variety Record, *S. tuberosum* variety *Centifolia* and *S. leptostigma*. All the hybrid seedlings proved to be tetraploid ($2n=48$), which has led to the conclusion that 24-chromosome gametes were acting from both sides; this hypothesis was supported by the intermediate characters of the hybrids.

In 1937 and from 1940 to 1943 *S. maglia* was grown in Western Pamir, 6 km from the town Khorog at an altitude of 2320 m. The climate of Khorog is a very dry and continental one; there are no rains during the summer, the relative humidity of air is low (down to 30—40 per cent and at some days to 20 per cent, especially in day-time) the temperature amplitude is wide, the temperature of the hight being low, especially beginning by the later half of August (about 0° and down to —2° and even —5°).

In 1937 only a single plant of *S. maglia* was grown which produced 15 seed balls; 5 of them contained no seeds. The remaining 10 seed balls contained a total of 49 seeds, only 13 of which deserved planting; the remaining were poorly filled or empty. In 1938 these seeds were planted at the Leningrad Experiment Station of VIR, and diploid seedlings were obtained from them possessing chromosome somatic numbers approximately 24, namely, $2n=25$, 26 (4).

From 1940 to 1943 *S. maglia* was yearly grown at a number of 25—50 plants at Pamir; every year there was abundant seed balls setting. About 70—80 per cent of the seed balls contained from 5 to 16 good seeds each. All the seedlings obtained from

the seeds of 1941—42 and raised at Pamir in 1942—43 also proved to be diploids with a haploid chromosome number $n=12$, 13. The occurrence of diploid seedlings among the progenies of the triploid *S. maglia* should beexplained by parthenogenetic development of the gametes containing 24 chromosomes or approximating numbers. Such gametes are evidently the most vital ones in *S. maglia*, which is indicated by the above instances of the functioning of the 24-chromosome gametes in hybridization of this species with tetraploid species, as well as by the observations by G. N. Nepli who has found in Pamir a considerable number of gametes with 24, 25 and 26 chromosomes in meiosis of the mother pollen cells of the triploid *S. maglia*. Parthenogenesis could be stimulated by the combination of the peculiar climatic conditions of Pamir, *viz.* low night temperature at the period of seed ball setting, dry air, the intensity of solar radiation and high content of ultra-violet and especially of red and infra-red rays.

During the last years at Pamir there was conducted a detailed study of the behaviour of the characters of the sexual progeny of the triploid *S. maglia* and in subsequent generations of the diploid strain of the same species.

The triploid maternal plant is characterized by the following characters: the plant is robust, the stem branched and abundantly flowering, of violet coloration and supplied with narrow saw-touthed wings; leaves slightly dissected, having 2—3 pairs of leaflets and 1—2 interstitial little leaflets with a large apical leaflet; corolla white, large, up to 3.5 cm in diameter, having wide lobes and very short mucrones; calyx light green with short mucrones; anthers yellow, spread, containing a small amount of normal pollen (15—30 per cent); stigma twofold; ovary quadrangular; seed balls spherical, light-green, dull, containing a few seeds (from 1 to 16); production of tubers scanty (from 1 to 12 tubers per plant).

The diploid plants S_1, *i. e.* plants obtained from the seeds of the triploid *S. maglia*, repeat the maternal triploid in their basic morphological characters: saw-toothed wings of the stem, slightly dissected leaves, white flowers, descending tissue of the anthers on the stamen filament, double structure of the stigma, quadrangular shape of the ovary, and spherical dull seed balls. Along with these similarities some differences were noted, *viz*, presence in some of the seedlings of a fourth pair of the leaflets, broad or narrow lanceolate shape of the leaflets against the broad ovoid shape in the triploid; further the points of the corolla were slightly expressed in the majority of the seedlings, and the mucrones of the calyx were elongated. The chief differences of the diploids consisted, however, in quantitative characters:the height of the plant was 20—30 cm against 50—60 cm, the length of the leaf 4.0—4.5 cm against 6.0—6.5 cm, diameter of the corolla 1.9 cm against 3.5 cm, the length of the anther 0.27 cm against 0.55 cm, the diameter of a seed ball 1.5 cm against 2.0—2.5 cm, *etc.* Besides, the diploid seedlings were distinguished by their increased fertility, by a high content of normal pollen (up to 70—90 per cent against 15—30 per cent), by a high number of seeds per seed ball (up to 70—80 seeds) and by a very high yield of tubers (up to 50—70 tubers per plant).

Plants S_2 obtained from the seeds of the diploid strain S_1, as well as the subsequent generation S_3 from the seeds of S_2, differ but scarcely from the S_1 plants. These plants, according to determination made by Nepli, had haploid chromosome numbers $n=12$, $n=13$. They similarly possessed high fertility, great proportion of normal pollen (80—90 per cent), produced numerous seeds per fruit (60—80 seeds) and abundant tubers (40—60 per plant). In all their quantitative morphological characters the S_2 and S_3 plants aproach the S_1 plants. Only in a few seedlings there were noted deviations in some characters, *e. g.* occurrence of a light-blue coloration of the corolla and presence of two extra pairs of leaflets and even interstitial leaflets.

Repetition in the S_1, S_2 and S_3 plants of the basic qualitative morphological characters of the triploid mother supports the hypothesis of the autotriploid nature of *S. maglia*, at the same time it makes one think that the diploid karyological strain is the original species which gave rise to the autotriploid *S. maglia*.

The constancy within three generations not only of the quantitative morphological characters but also of the physiological properties, such as high fertility and

abundant tuber-production, points to another genotypic structure of the diploid strain, and makes it possible to consider it a true species which formerly occupied an area in Chile.

The experimental production of the original species of the Chilian autotriploid is of interest not only from the point of view of the origin or evolution of *S. maglia*, but as well from the point of view of the secondary form production under the natural conditions of the mountainous regions of the Western Pamir. Of special interest is therefore the study at Pamir of the sexual offspring of other autotriploid species, like the Uruguaian *S. Commersonii* and the Argentinin *S. Millanii*.

Allotriploid forms in the Western Pamir also give rise to new karyological strains but of polyploid nature. In the offspring of the Mexican *S. Vallis Mexici* were obtained hexaploid plants (5), and in the progeny of the cultivated mountainous forms from Andes, *S. tenuifilamentum*, *S. mamilliferum* and *S. cuencanum*, there were found tetraploids (6).

Western Pamir is consequently of outstanding interest as an mountainous region of secondary evolution of the South American potatoe.

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REFERENCES

[1] С. М. Букасов, Изв. Всес. геогр. об-ва, 2 (1940). [2] С. М. Букасов, Вестн. соц. раст., № 1 (1941). [3] Р. Л. Перлова, Вестн. соц. раст., № 4 (1940). [4] R. L. Perlova, C. R. Acad. Sci. URSS, XXVII, No. 7 (1940). [5] R. L. Perlova, ibid., XXV, No. 5 (1939). [6] R. L. Perlova, ibid., XXVII, No. 1 (1940).

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ECOLOGY

INFLUENCE OF SALINITY ON THE EGGS OF *COREGONUS MIGRATORIUS GEORGI*

By V. I. OLIPHAN

(Communicated by I. I. Schmalhausen, Member of the Academy, 8.VIII.1944)

Experimental observations on the reaction of fishes at early stages of development to the salinity of the environment, carried on by the author for several years ([1],[2]) gave the following results:

For the representatives of *Cyprinidae*: *Abramis brama*, *Cyprinus carpio*, *Rutilus rutilus caspicus*, *Rutilus rutilus heckeli*, *Scardinius erytrophtalmus*, and of *Percidae*: *Lucioperca lucioperca*, *Acerina cernua*, *Perca fluviatilis*, a salinity of 10 per mille is the upper sublethal limit at the stage of embryonic development, whereas the larvae of these fishes survive and develop normally at salinities below 7.5 per mille only, *i. e.* in a medium with a lower osmotic pressure than the blood pressure of the fish.

A similar sensitivity to salinity was observed in *Acipenser stellatus*, a representative of the *Acipenseridae*.

Representatives of the genus *Caspialosa* (fam. *Clupeidae*), *Caspialosa volgensis* ([1]) and *Caspialosa caspia* ([3],[4]), who breed in fresh water, differ greatly from the above fishes in their behaviour in respect to salinity. Their eggs and larvae survive and develop normally at salinities with osmotic pressure higher than the pressure of the internal medium of the fish.

The different behaviour of the two *Clupeidae*—representatives of true marine fish—as compared with the nine representatives of species belonging to families of fresh-water origin, permits to suggest that the sensitivity to salinity may be related to the origin of the fish.

In 1942, while engaged in ecological and physiological investigations at the Burjat-Mongol piscicultural station, on eggs, larvae and fry of *Coregonus migratorius*, the author investigated also the influence of salinity on the fertilization and development of eggs of this fish*.

The experimental procedure was the same as in our previous investigations. As medium were used different dilutions of Ringer's solution, which in our earlier experiments gave identical results with sea-water of similar salinity.

Fertilization of the Eggs of *Coregonus migratorius* at Different Salinities. Small numbers of mature running eggs, obtained from a spawner caught on the spawning grounds, were transferred to six dry glass beakers. The roes of a mature male were added to the eggs, and in five of the beakers was ad-

* Thanks are due to the Director of the Station A. N. Namolov for affording us possibilities to carry on our work, as well as to M. K. Emme for help in experimenting.

ded some Ringer solution with a salinity of 2.5, 5, 10, 15 and 20 per mille; to the sixth beaker river-water was added. The eggs and roes were mixed with the aid of a feather, allowed to stand for a few minutes and washed with several lots of the solution to remove the viscid substance. Two hundred eggs were taken from each beaker for further observations, and kept at salinities identical with those used in fertilization and washing.

The eggs developed at all salinities, except 20 per mille, at which evidently no fertilization had taken place.

Survival and Development of Eggs of *Coregonus migratorius* at Different Salinities. Two series of experiments were carried out, the first on the influence of salinities of 2.5, 5, 7.5, 10, 12.5, 15 and 17.5 per mille and the second—of 5, 10 and 15 per mille; each series involved 200 eggs.

At a salinity of 17.5 and 15 per mille all the eggs perished on the 5th day after fertilization, when normally developing eggs were already of the gastrulation stage. At 12.5 per mille mortality was high, reaching 100 per cent of the later stages of development.

The experiments were finished after 20 days, at the stage of formation and growth of the embryos; the heads of the embryos were markedly thickened owing to the growth of brain and eyes; the crystalline lenses in the expanded eye-cups were quite noticeable; the segmentation of body and tail was well expressed, the tail detached from the yolk and rather long.

The results of these experiments are summarized in the table.

Survival of Eggs of *Coregonus migratorius* at Different Salinity

Series No.	Salinity, per mille	2.5	5	7.5	10	12.5	15	Control (river-water)
I	Survival, %	80.0	78.5	72.5	42.5	0	0	69.0
II	» »	—	86.3	—	64.6	—	0	89.1

In both experiments an increase of mortality at a salinity of 10 per mille was observed only at the stage of gastrulation; the mortality still increased at the stage of embryo formation, and a considerable number of eggs was found to develop abnormally.

Conclusions. 1. In eggs of *Coregonus migratorius* fertilization can take place at salinities up to 15 per mille.

2. Normal development and survival of the fertilized eggs takes place at salinities below 10 per mille. At 10 per mille a sharp increase of mortality is observed, along with a considerable number of abnormally developing embryos.

3. The upper limit of salinity for fertilized eggs of *Coregonus migratorius* is very near to the limit found for the fertilized eggs of *Cyprinidae Percidae* and *Acipenseridae*.

Discussion. In literature we have found only one indication to the reaction to salinity of the eggs of *Salmonidae*, *viz.* the experiments of Nordgard ([5]) with sea salmon, whose fertilized eggs developed normaly only at salinities below 9 per mille.

Referring to the experiments of Nordgard and the investigations of Cherniavin([6]) demonstrating that the evolution of *Salmonidae* proceeded in connexion with fresh water (few marine anadromous forms were re-adapted to marine life), we mentioned ([1]) that the example of salmon eggs confirms our assumption as to relation existing between the origin of fish and their reaction to salinity at early stages of development.

This suggestion is in full agreement with the results of the experiments described above with eggs of *Coregonus migratorius*, related as to the origin to *Corego-*

nus automnalis of the North Poltar Sea. This fish enters the Siberian rivers Yenissei, Lena and others on its autumnal spawning migrations.

In November 1942 were published the results of the experiments by Misharin (7) who investigated the influence of salinity upon the fry of some Baikal fishes. It may seem at first sight that the data of these experiments are in contradiction with our results.

In the experiments of Misharin, the fry of *Coregonus migratorius* at the age of 1.5 months survived normally a salinity of 15.45 per mille, whereas the fry of *Cottocomephorus grewingkii* perished already at 6.18 and 3.09 per mille; the fry of *Phoxinus phoxinus* could not resist to these concentrations either. Referring to the results of his experiments, Misharin says that «experiments of this kind may shed light upon the origin of the Baikal animals».

We cannot consider the salt-resistance in 1.5-month-old fry of *Coregonus migratorius* as a proof of the marine origin of this species.

Our experiments with 1—2 month-old fry of *Abramis brama*, *Rutilus rutilus caspicus*, *Cyprinus carpio* (2) and *Acipenser stellatus* have shown that at this age they are highly resistant to the salinity of the environmental water. In our experiments they were easily adopted to sea-water with a salinity of 15 per mille, though they all belonged to families of fresh water origin and were very sensitive to much lower salt concentrations during their embryonic and, especially, larval development.

It should also be noted that the different sensitivity to salinity in the fry of *Coregonus migratorius*, on the one hand, and of the fry of *Cottocomephorus grewingkii* and *Phoxinus phoxinus*, on the other, is evidently partly due to age differences. To judge from the data of the experiments of Misharin, the author's material consisted of larvae of *Cottocomephorus grewingkii* rather than of fry, as in the case of *Coregonus migratorius*.

Received
8.VIII.1944.

REFERENCES

[1] В. И. Олифан, Зоол. журн., XIX, в. 1 (1940). [2] В. И. Олифан, Труды Всес. н.-и. ин-та мор. рыбн. хоз-ва и океанографии (ВНИРО), XVI (1941). [3] И. А. Бородин, Вестн. рыбопром., XIII, № 6 (1898). [4] Н. С. Строганов, Уч. зап. МГУ, 33, (1939). [5] Nordgaard, Mitt. d. Westpreuss. Fisch. Ver., XII, No. 1 (1900). [6] В. В. Чернавин, Журн. петрогр. агроном. ин-та, № 3—4 (1921). [7] К. И. Мишарин, Тр. Вост.-Сиб. гос. ун-та, II, № 3, Биология (1942).

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PLANT PHYSIOLOGY

STUDIES ON THE PHYSIOLOGY OF THE GERMINATION OF POLLEN

I. MUTUAL STIMULATION IN THE GERMINATION OF POLLEN GRAINS

By I. N. HOLUBINSKY

(Communicated by A. A. Richter, Member of the Academy, 20.VIII.1944)

In 1944 for some reasons so far unknown, the pollen of the majority of Angiosperms studied showed an extremely poor germination in artificial media. For example, the pollen of various *Digitalis* species, which usually germinate very well in sugar solutions practically of any concentration, showed in 1944 a considerably lowered germination percentage and developed short pollen tubes of the kind quite unusual in this genus (1).

In our attempts to discover the cause of this phenomenon we repeatedly germinated a great number of samples: it was found from these experiments that both the length of the pollen tubes and the proportion of germinated pollen grains were considerably increased with the density of the sowing of pollen. The difference appeared to be very significant. A similar phenomenon, *i. e.* the mutual stimulation of the pollen grains germinating in dense sowing, was observed by other writers (3,4).

In order to study this problem in more detail we arranged special experiments on the mutual stimulation of the germinating pollen grains in a number of species belonging to different families.

A fifteen per cent solution of saccharose, in distilled water, served as the medium for germinating pollen. The concentration remained the same for all the species tested. Neither agar nor gelatine were added, as we considered it unnecessary to adjust the medium to each particular species.

Germination was conducted in a hanging drop within a Petri dish. Each experiment was repeated twice. Six drops of sugar solution were placed into the Petri dish, two for each set: two drops containing a small amount of pollen (of the order of several dozens of pollen grains per drop), two drops containing a medium amount (several hundreds of pollen grains), and two drops with a great amount (several thousands) of pollen. The dishes were placed on a table in a light laboratory room (direct sunshine being excluded); the temperature was 22—24 °C. Records of the proportion of germinated pollen grains and of the length of the pollen tubes were taken 20—24 hours after sowing. The pollen tubes were measured on preliminary drawings made with the aid of Abbe's *camera lucida*. The germination percentage was found from counts of the total number of the pollen grains and the number of the germinated ones within the microscope field.

Care was given to select the experimental material from different families, in order to find out whether the regularities observed were of general nature.

From a total of over 30 species in 15 of them we succeeded to induce germination; the results are summarized in the table.

As may be seen from the table, in all the species studied there exists a clear correlation between the amount of pollen contained in a drop of sugar solution, and the germination per cent and the length of the pollen tubes. Not a single species

Results of Germination of Pollen with Varying Density of Sowing

Species	Results of germination					
	Small amount of pollen		Medium amount of pollen		Large amount of pollen	
	germination %	average length of pollen tubes	germination %	average length of pollen tubes	germination %	average length of pollen tubes
Humulus Lupulus L.	1.5	80	14.5	107	13.5	133
Papaver somniferum L.	0.0	—	21.5	75	34.5	89
Rubus caesius L.	85.0	165	96.5	228	98.0	436
Medicago sativa L.	37.0	193	89.5	547	98.0	867
Tropaeolum majus L.	84.5	213	91.0	410	92.5	508
Hypericum perforatum L.	58.5	170	78.5	215	86.5	250
Oenothera biennis L.	27.5	454	32.0	530	49.0	712
Datura stramonium L.	0.0	—	0.0	—	5.5	52
Lycopersicum esculentum Mill.	38.0	156	49.5	295	51.0	314
Linaria vulgaris Mill.	17.0	97	49.5	290	58.0	527
Digitalis ambigua Murr.	3.0	156	51.5	291	68.0	413
Digitalis purpurea L.	9.5	134	57.0	260	73.5	335
Campanula persicifolia L.	56.0	269	88.5	549	95.5	556
Helianthus annuus L.	6.5	28	11.0	32	12.5	32
Tanacetum vulgare L.	4.5	32	8.0	34	7.5	36

did show the reverse relation, or even a lack of stimulation by the increase of density. The pollen of some of the species has poorly germinated in the medium selected by us, so that the effect of density was but slightly observable (representatives of *Compositae—Helianthus annuus* L. and *Tanacetum vulgare* L.); the majority of species showed a sharp increase both in germination per cent and in lenght of the pollen tubes. The mutual stimulation of germination was especially manifest in the case of *Digitalis* spsp., *Medicago sativa*, *Linaria vulgaris*, *etc.* The pollen of *Datura stramonium* did not germinate unless the amount of pollen in a drop of the medium was a large one.

The limit of density of sowing pollen grains in artificial media could not be found from our studies. We tried to place into a drop of sugar solution whole heaps of pollen (the number of pollen grains being so great as to render any count impracticable), and observed in such cases an especially strong growth of the pollen tubes.

Taking into account that the stimulation of germination with the increase in density was demonstrated by us in 15 species belonging to 14 genera and 11 families, we feel justified to conclude as to the general nature of the regularity discovered and to assume that such a stimulation must exist in all the Angiosperms.

Once established the fact of stimulation of pollen grains germinating in artificial media, there is full ground to expect that a similar stimulation will exist in the process of germinating on the stigmata. One should consequently not content oneself with minimum amount of pollen for plant pollination. The number of pollen grains should hundreds times exceed that of the ovules to be fertilized.

Similar phenomena are observed in the animal husbandry in artificial insemination. The so-called «microdoses» of sperm (which are necessary for a successful insemination of the animals), containing numbers of sperms which exceed many times that needed arithmetically (2), should be considered to support our conclusions.

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REFERENCES

[1] И. Н. Голубинский, Бот. журн. СССР, XXII, № 6 (1937). [2] В. К. Милованов, Яровизация, № 2 (1941). [3] R. Savelli, C. R., **210**, No. 15 (1940). [4] R. Savelli et C. Caruso, C. R., **210**, No. 5 (1940).

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PLANT PHYSIOLOGY

INTERACTION BETWEEN ROOTS AND SOIL COLLOIDS AS A PROBLEM OF MINERAL NUTRITION OF PLANTS

ROLE OF THE ROOT SYSTEM

By E. I. RATNER

(Communicated by A. N. Bach, Member of the Academy, 22.XI.1944)

The power of the plant to retain the cations absorbed by it from their desorption by soil decreases from the above-ground organs to the root system ([1],[2]). The low retaining power of the root fully agrees with their rôle of an organ supplying the above-ground organs of the plant. Under favourable conditions of supply with mineral elements, this property of the root system is responsible for an easy reflux of absorbed water and mineral substances to the above-ground organs. If, however, in their external medium the roots encounter some mechanism competing with them in binding definite cations, as the colloids of the soil, then the poor retaining capacity of the roots may become a factor impeding the supply of the plant with their very cations. Under definite conditions it may even result in an inverse movement of the cations, *viz.* instead of the direction soil—plant, there will be a migration of cations in the direction plant—soil.

Under natural conditions, this relation between the above-ground organs of the plant and the soil colloids lays an imprint upon the whole life of the root system as an organ of supply with mineral substances and largely with water, too.

Our numerous experiments went to demonstrate ([2-4]) that the soil colloids assume the rôle of an outspoken competitor to the plant in the fixing of cations in case they are highly saturated with sodium. In order to be able to consider this phenomenon as being due to the competition between two colloidal systems, *viz.* the soil colloids and the colloids of the vegetable cells, it was necessary to make out the difference between the effect of Na upon cation exchange in the plant when supplied to the external medium in a colloid soil complex, or in the form of soluble salt. The method of severed root applied by the American investigators seemed to us scarcely appropriate to the solution of this problem, for it fails to shed light upon the essence of the changes that take place in the relations between the roots and the above-ground organs of the plant, leading therefore to contradictory results.

The experiments here described were carried out with barley of the variety «Wiener 01163», grown in water cultures upon Helriegel's solution prepared with tap-water, and taken into experiment when 13 days old. Previous to the experiment, the nutrient solution was renewed twice. After their roots had been well rinced in distilled water to remove the traces of the nutrient solution, the plants were immersed in the following media: 1) suspension of soil (100 g to 800 cm^3 of water) containing among its exchange cations 10 m-eq. Na, 3.6 m-eq. Ca and 3.9 m-eq. Mg; the soil was saturated with these cations up to a pH of about 7.2; 2) NaCl solution (800 cm^3 of water) containing 10 m-eq. of sodium; 3) $NaCl + CaCl_2 + MgCl_2$ solution

containing various amounts of Na, Ca or Mg cations, according to the experimental set; and 4) 800 cm^3 of distilled water.

Each vessel was made to contain 50 plants, the experiment was run at two-fold replicaton. It was composed of two sets. In one set the interaction between the roots and the respective media was made to last no more than 3 minutes, after which the roots were washed with distilled water, the above-ground organs severed from the roots, and both roots and above-ground part were dried and subjected to analysis. In another experimental set the interaction between the roots and the respective media was made to last 23 hours, under regular blowing-through of air, practized every hour during 10—15 minutes (during the night the procedure was suspended).

Table 1

Experimental set	Dry weight of plant in g per vessel		K content in plants in mg per vessel		
	roots	above-ground organs	roots	above-ground organs	total
Plants kept during 3 min. in:					
Distilled water (control)	0.78	2.20	21.1	110.5	131.6
Soil suspension	0.81	2.31	13.7	110.8	124.5
NaCl solution	0.80	2.29	16.4	109.3	125.7
$NaCl+CaCl_2+MgCl_2$ solution	0.79	2.19	18.0	110.5	128.5

Table 1 represents the results of the fisrt experimental set. From this table it may be seen that a three minute-long interaction between root and soil suspension proved sufficient to bring about a well-pronounced desorption of potassium from the roots. A similar effect, though at a less pronounced degree, may be observed in the case of interaction between the roots and soluble salts. We thus see that an experiment in which the exchange phenomena involve the root system alone, without affecting the above-ground organs of the plant, fails to reveal the specific traits of the colloid soil complex, as of a possible competitor to the plant.

Table 2

Experimental set	Dry weight of plants			K content in mg per vessel			Ca content in mg per vessel		
	roots	above-ground organs		roots	above-ground organs	total	roots	above-ground organs	total
		in g per vessel	in % of control						
Prior to experiment (control)*	0.78	2.20	100	21.1	110.5	131.6	19.5	23.0	42.5
After being kept for 23 hours:									
In distilled water	0.78	2.64	120	16.7	117.9	134.6	13.4	25.5	38.9
In soil suspension	0.79	2.63	120	11.7	101.0	112.7	8.2	19.0	27.2
In NaCl solution	0.79	2.59	117	13.2	117.7	130.9	13.4	23.9	37.3
In $NaCl+CaCl_2+MgCl_2$ solution	0.76	2.57	117	—	—	—	17.8	24.7	42.5

*As control served plants kept during 3 minutes in distilled water.

Table 2 gives the results of the second experimental set.

Similar to their losses in Ca, the losses in K by the roots are very high as compared to the control. The character of these losses differs greatly, however, according to whether the roots are made to interact with salt solutions or into soil suspension.

In the former case, similar to the case of distilled water, the total potassium lost by the roots is found to have migrated to the above-ground organs which within the 23 hours of the experiment were found to have increased by 17—20 per cent by weight (the experiment was carried out in the light). We thus see that within this period the plants re-absorbed also the potassium which had been lost by the roots during the first minutes of interaction between them and the NaCl solution. The absorbing power of the intensely growing sprouts proved a much more powerful agent than the displacing effect of the sodium ion from the NaCl solution. A similar picture is observed in the case of Ca, save for that the losses of the roots in this element are to a certain degree caused by its migration to the above-ground organs, while part of it is eliminated into the external medium. The latter circumstance is to be explained by the fact that Ca is accumulated in the above-ground organs of the young plants at a much lower rate than K(2).

Quite different is the picture observed in the case of interaction between the roots and soil suspension. In this case a loss of K and Ca not only by roots, but also by the above-ground organs is revealed by analysis, in spite of the intense growth which takes place in either of them.

Thus, the specific nature of soil colloids as possible competitors to the plant in fixing the cations is made evident by such experiments only where the reaction of interaction is made to involve not the roots alone, but the above-ground organs of the plant as well.

We were successful in tracing in time the process of migration of cations from the above-ground organs to the external medium in the case of interaction between the roots and the soil colloids in another experiment, too; this experiment was carried out with the winter-variety of wheat «Lutescens 329», under conditions approaching those of the experiments described above; the plants were taken for the experiment when 27 days of age. Previous to experiment they were kept upon Helriegel's solution made with tap-water; the solution was renewed once. The results of this experiment are summarized in Table 3.

Table 3

Experimental set	Dry weight of plant in g per vessel		Ca content in mg per vessel		
	roots	above-ground organs	roots	above-ground organs	total
Plants kept for 36 hours in distilled water (control)	2.03	9.00	14.2	58.7	72.9
Plants kept on soil suspension saturated with a 30% Ca+70% Na solution:					
During 3 minutes	2.06	8.66	5.4	57.0	62.4
During 36 hours	1.93	8.82	8.5	44.9	53.4

From Table 3 it may be seen that the first minutes following the interaction between the roots and the soil suspension are characterized by a pronounced loss of calcium by the roots, which does not yet affect the above-ground organs. However, as a result of a sharply disturbed equilibrium in the calcium distribution which sets in between the above-ground organs and the roots, an inverse flow of this cation from the above-ground organs to the external medium is soon established. The calcium contents in the roots is even found somewhat to increase as compared

with what is observed after the very first minutes of interaction between roots and soil suspension.

It is of interest to note that under definite conditions it is possible to observe simultaneously an upward flow of a given cation, say, of calcium, through a definite part of the root system, put under conditions favouring the absorption of this cation by the plant from soil (high saturation of the colloidal soil complex with calcium), and a downward flow of the same cation along another part of the root system, where the conditions are favourable for the desorption of this cation by the soil from the plant (low concentration of calcium in the soil, along with a high saturation with Na). These conditions may take place in the case the different parts of the root system are situated at different horizons of solonetz soil, which may possibly account for the ameliorative effect of live plants upon these soils ([2]).

As shown by a further analysis of roots under this experiment a detailed description of which is to be found elsewhere, the phenomenon here discussed is due to a considerable difference between the calcium concentration of the part of the root system which was in contact with the readily accessible source of that cation (soil high in Ca) and the part of the root system which was in contact with a soil low in Ca and high in Na. As a result, the one part of the root system played the part of an agent supplying the above-ground organs of the plant with calcium, whereas another part of the root system was acting in an opposite direction, transferring the calcium taken off the above-ground organs to the more powerful acceptor, which in this case was the soil with a high Na saturation. In this way the plant was found able within a definite range to transfer Ca from one soil to another.

The phenomenon here described permits of an analogy with the phenomenon observed by Breazeale ([5]) and Chizhov ([6]) who have found that plants are capable of transferring water to dry soil in the case where one part of their root system is situated in dry soil, while another is situated in a medium rich in readily available water. This phenomenon can possibly be interpreted also as a result of competition between the soil colloids and the root system in their water-fixing activity; as to the competition between the separate parts of the root system and the above-ground organs, its rôle in this process can be characterized as that of an intermediate link.

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Received
28.XI.1944.

REFERENCES

[1] E. I. Ratner, C. R. Acad. Sci. URSS, XLIII, No. 3 (1944). [2] Е. И. Ратнер, Почвоведение, No. 4—5 (1944). [3] E. I. Ratner, Bull. Acad. Sci. URSS, série biol., No. 5—6, 1153 (1938). [4] E. I. Ratner, C. R. Acad. Sci. URSS, XLII, No. 7 (1944). [5] J. F. Breazeale, Univ. of Arizona, Agr. Exp. Stat. Bull. No. 29 (1930). [6] B. A. Chizhov, Bull. Acad. Sci. URSS, série biol., No. 4, 555 (1940).

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EXPERIMENTAL MORPHOLOGY

REGULATION PROCESSES IN THE METAMORPHOSIS OF INSECTS

SELF-DIFFERENTIATION OF THE WING IN BUTTERFLIES

By D. M. STEINBERG

(Communicated by I. I. Schmalhausen, Member of the Academy, 2.I.1945)

As has already been shown in our preceding papers (2), the lateral hypodermal parts of the thoracic segments in *Galleria melonella* caterpillars gives rise to the embryonic territory of the wing, which is the source of its regeneration throughout the period of the development of the larvae.

The most convenient method for studying the possibility of self-differentiation of the wing from the embryonic territory is by grafting the latter to one of the abdominal segments, *i. e.* into a region removed far enough from the original place of the transplant. For this purpose the hypoderm together with the imaginal disc was excised; the imaginal disc was separated in Ringer solution, and the hypoderm without the imaginal disc was grafted to the lateral surface of one of the abdominal segments, usually the 3th or 4th segment. The transplantation was preceded by the removal of a corresponding piece of abdominal hypoderm, instead of which a bit of thoracic hypoderm was to be transplanted.

The transplantations were mostly carried out on a single caterpillar specimen. The different series of the experiments varied as to the distance of the place of transplantation from the media-dorsal line. Besides, the orientation of the transplant with respect to the longitudinal axis of the body also varied in the different series of the experiments. In a large part of the experiments the orientation remained unchanged. In some experiments, however, the hypoderm transplanted was turned through 180°. The operations were performed at the two latest caterpillar stages.

In a number of cases the embryonic territory transplanted gave rise to the formation of an imaginal bud. During the pupation the disc was evaginated forming the wing rudiment of the butterfly. The adjoined table gives an idea of the number of the wings developed in the abdominal region of the pupa; only successful experiments, in which the caterpillars pupated normally, are given in the table.

As will be seen from the table, the number of the pupae, in which an extra wing developed from the embryonic territory implanted, is not high, making up no more than 11.2 per cent to the total number of the pupae operated. The highest number of successful operations (21.2 per cent) was obtained in the case of transplantation of the territory of the last caterpillar instar at the level of the stigma of the abdominal segment, *i. e.* in the case where the original location of the graft in the segment in dorso-ventral direction was preserved, the imaginal disc of the wing being located at the very place where in the other segments the stigma is located.

The development of a wing from an embryonic territory transplanted nearer the dorsal line, *i. e.* somewhat farther from the stigma, was observed in 9. 4 per cent of cases only. The wings failed to develop at all if the bit transplanted was turned

Number of Grafts of Embryonic Territory to Abdominal Segments

Stage (instar)	Number of transplants with normal orientation of material transplanted				Number of transplants turned through 180°		Total	
	at the level of stigma		near to dorsal line					
	regeneration took place	no regeneration	regeneration took place	no regeneration	regeneration took place	no regeneration	regeneration took place	no regeneration
Last caterpillar instar . . .	11	41	3	29	—	33	14	103
Last but one caterpillar instar . .	1	10	—	6	—	—	1	16
Total . .	12	51	3	35	—	33	15	119

through 180°. When the transplantation was made at the last but one stage, the development of the wing was observed in a single case only. However, this observation can hardly give right to draw any conclusion, first, because of the insufficient number of observations, and, secondly, because of the fact that in this single case there was observed not only a most complete development in the butterfly of the pupa wing, but of an extra wing also.

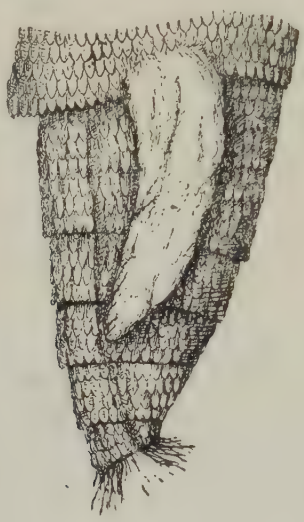
In all 14 caterpillars operated upon at their last stage the imaginal discs were found to be evaginated and spread to some degree, being normally oriented in a caudal direction. On the average, the wings reached the middle of the next segment. There was no reason to expect a more complete spreading of the wing, since the development was brought about under unusual surroundings, and the blood pressure which plays an essential part in the normal development of the wing did not take place in the cases under consideration.

The surface of the evaginated disc was covered with chitin, forming folds and furrows as a result of incomplete spreading. If we imagin these folds duly spread, the wing developed in the abdominal segments will not differ as to its size from the wing of the third class, developing in the thorax upon the removal of a significant part of the embryonic territory (3).

The single butterfly (No. 941) which developed from the caterpillar operated at the last but one stage, was found to possess an evaginated wing developed to a more considerable degree than in the remaining butterflies. The wing of this specimen grew to two extra-segments, and the numerous folds point to that the real size of the wing would be considerably greater if normally spread.

Cross-sections through the evaginated wing on the abdominal segments have shown that the hypoderm cells underwent histological changes similar to those of the normal wing. The basal membrane became indistinguishable;the cell basements were stretched out into long protoplasmic filaments which formed anastomoses, thus connecting directly the epithelium of the superior and inferior surface of the wing. The nuclei of the cells were at different levels. Sinuses were formed among the cell basements similar to those of the normal wing. The sinuses were filled with blood almost lacking cellular elements. Only isolated blood cells were met within the section among the protoplasmic filaments of the epithelium. In the distal part of the wing the epithelium was more flattened. The nuclei were set wider apart from one another. Only fine protoplasmic fibres were found to connect the epithelium of the superior and inferior layers.

The histological differentiation in a two-day-old pupa in general does not differ from what has been described above; the only difference is that the basal membrane becomes still less distinguishable. As well-known, in a normal wing the tracheal tubules pass through the blood sinuses. The regenerates of the abdominal segments are either lacking the tracheal tubules, or else these are represented by a single branch. Thus, the presence of a trachea is no necessary condition for wing evagination; it is purely casual, depending probably upon the location of the regenerate: the nearer to the stigma, *i. e.* to the abdominal tracheal column, the more probable is the existence of tracheal branches, growing into the regenerate.



The histological differentiation of the wing epithelium of the regenerate is somewhat behind that of epithelium under normal development conditions. Wing evagination seems therefore to occur somewhat prematurely. At any rate, regeneration causes a regulative lagging of pupation, similar to that described by me for wing regeneration at an appropriate place of the thoracic segments (3). This is especially clear in the cases where the specimens operated are lacking thoracic regenerates, having only an abdominal regenerate developing from the embryonic territory transplanted. For example, specimen No. 1046 pupated on the 13th day after the beginning of cocoon formation, while under normal conditions this process takes place on the 3rd day of pupal development.

Out of the 15 regenerates obtained, 14 were fixed, or died at the pupal stage. The fact is that as the pupal development proceeds, the abdominal regenerate becomes gradually dry. The development of the regenerate at an unusual place, the absence of normal blood circulation, a total or nearly total absence of tracheal tubules, and a deficient aeration resulting from this fact prevent in a large number of cases any further differentiation of the wing, and probably are the cause of retardation of development during the first days of pupal development.

Only one specimen (No. 941) was found to have an extra abdominal wing (see figure), 2.1 mm in size. The numerous folds did not allow to determine its real shape and to answer the question whether it was an anterior or a posterior one. The surface of the wing was nearly destitute of scales. The whole wing contained a total of 23 scales only. The scales were normally developed, differing from the normal ones neither in size nor shape. Among them it was possible to discriminate between covering medial and edge scales, fibre-like, as well as those with fibre-like basement but wider at the end. Among the covering scales two were black, and two reddish in colour. Such scales are met on the upper wing of butterfly only. The scales which had developed from the embryonic territory of the mesothorax

thus corresponded to their origin. The histological differentiation of the epithelium thus comes to its normal end, leading to a formation of the scales irrespective of the location of the regenerate. Their small number depends upon the insufficient supply of nutrient substances and deficient aeration. In his experiments on the transplantation of the hypoderm pieces into the fat-body, which led to a formation of isolated epithelial vesicles, Piepho ([1]) also observed self-differentiation of epitholium, leading ultimately to a development of normal scales.

Thus, experiments involving grafts of the embryonic territory of the wing have shown that the cells of the territory are capable to preserve developmental potencies and to give rise to the wing formation independently of their surroundings, the developing regenerate showing a normal histological differentiation corresponding to the origin of the embryo-gerenerative territory.

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REFERENCES

[1] H. Piepho, Biol. Zbl., 58, 7/8 (1938). [2] Д. М. Штейнберг, Биол. журн., 7, 5/6 (1938). [3] D. M. Steinberg, Bull. Acad. Sci. URSS, sër. biol., No. 4 (1939).

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ZOOLOGY

ZOOGEOGRAPHY AND SOME MORPHOLOGICAL PECULIARITIES OF THE FAMILY *TRIGONALOIDAE (HYMENOPTERA)*

By V. V. POPOV

(Communicated by E. N. Pavlovsky, Member of the Academy, 1.IX.1944)

The tropical character of the family *Trigonaloidae* is beyond any doubt. According to Bischoff's (1) review, two of the four subfamilies are purely tropical and widespread. The subfamily *Seminotinae* is also tropical, but its distribution is limited to South and Central America. Finally, the subfamily *Trigonaloidae* is the most widespread one, having its representatives in the Palaearctic and Nearctic regions. Out of the 75 species known up to the present, 9 species were found in the Palaearctic and 11 species in the Nearctic regions. However, 8 species out of this number were found in Japan and Korea, and 6 species in Mexico. Only a single species is known (*Pseudogonalos hahni* Spin.) to be widely spread nearly all over the palaearctic region. The same is true of North America. *Bareogonalos canadensis* Harrington, found in Canada and British Columbia, is obviously of southern and foreign origin. Two centres of the most intensive species formation may be pointed out at present. One of these is South- and Central American (including Mexico) comprising 26 species (or 35.1 per cent). The other is Oriental (including Malacca, Japan, and Korea). It comprises 30 species (or 40.6 per cent).

Information as to the parasitism of *Trigonaloidae* is rather scanty. Representatives of the subfamily *Trigonaloidae* are the hyperparasites (or secondary parasites) of parasitic *Ichneomonidae*. Representatives of the remaining three subfamilies are parasitic of various *Vespidae*. It is evident that *Trigonaloidae* are no homotopic parasites, and their geographical distribution is dependent not only on the distribution of the corresponding various hosts, but in the first place on abiotic factors. The fact that they are confined to humid, warm, forest districts is quite evident. According to Brues (2), the most primitive groups of *Hymenoptera*, not only subsisting on plants but also parasitic, are characterized by being confined to districts with woody vegetation.

From this point of view of interest is the northern boundary of distribution of *Pseudogonalos hahni* (Spin.), and the northern boundary of penetration of the elements of Oriental region into the Palaearctic one. The following data resulting from a study of the materials of the Zoological Institute of the Academy of Sciences of the USSR may contribute to the actual knowledge of this group.

1. *Bareogonalos jezoensis* (Uchida). The species was known from Japan (Uchida, 1929) and Java mountainous districts, at an altitude of 1200—1500 m above sea level (van der Vecht, 1933). At Java its hosts are *Vespa velutina* Lep. and. *V. analis* F. A new place of finding is the Soviet Harbor, the Maritime Province, VII, 1909, 1 ♀ (V. Arseniev). The species is closely related to *B. canadensis* Harrington (Canada, British Columbia) parasitic of *Vespa occidentalis* Cr. (*Vespula pennsylvanica* (Sauss.) after van der Vecht, 1933). The second and last

species of this genus and of the subfamily *Bareogonalinae* as a whole is *B. scutellaris* Cam., known from Mexico only.

2. *Nanogonalos mongolicus* sp. n. R. Sielchjin, Great Khingan, East Mongolia, 27.VIII.1899, 1 ♂ (V. Soldatov); Buin gol River, East Mongolia, 3.VIII.1899, 1 ♂ (V. Soldatov),

Body 7—8 mm length. Anterior wings 7 mm in length. Antenna consists of 25 segments. Segments 11 to 16 with tiloids. Left and right mandibulae have 3 and 4 teeth, respectively.

Head equal to the body width, densely punctured. The bright spaces between the clots vary from $^1/_3$ to 1 of their diameter. Scape of antennae densely and rather slightly punctured, bright. Body more or less bright, punctured more roughly and densely, nearly wrinkled. Body sides partly dull. Abdomen rather roughly, evenly punctured. Very bright spaces between the punctures are equal to $^1/_4$—1 of their diameter, or somewhat more. Tergit I without angular furrows, very bright and smooth. Apical parts of II and III sternites markedly flattened, sparingly punctured, bright. The apical non-punctured part of sternit III is narrower.

Black. Very narrow stripes along the outer orbit of the eyes, the wider and shorter on the inner orbit; two triangular spots, on the sides of the clypeus are yellow in colour. Antennae, beginning from the apex of scape, light-brown. Mandibulae reddish-brown nearly throughout. Anterior halves of the anterior wings dark. Stigma brownish-yellow. There are two yellow triangular spots on the sides near the anterior edge of mesothorax, humeral yellow tubercles and three transversal yellow spots nearly fused with one another on the metathorax. The apex apices of the trochanter II of the second pair of legs and trochanters I and II of the third pair of legs, yellow. The apex of femurs and tibia of the third pair are reddish. Tarsi dark-reddish. Middle subapical small spot of I tergit, the narrow subapical edge spots of the I sternit nearly fused with each other. Wide submarginal band of the II segment (markedly constricted in the middle of tergit and in the middle and at the edges of sternit and large transversely) and rounded spots of the tergits IV—VI yellow. Sternit VII widely rounded, nearly four times as short as wide; its basal lobes stretched and narrow. Sternit VIII angular, nearly stright along its anterior margin, with narrow and straight basal lobes and longer triangular middle lobe. Basal ring of genitalia well developed and rather long. Stipites wide, rather short, rounded and distally slightly constricted; their apical part separated, laterally flattened, and densely covered with short hairs. Sagittae narrow, nearly straight, rather long, widely rounded apically with rather short and solid basal lobes, slightly bent dorsally at the free end; volsellae ventrally included into stipites, and have well developed cuspis and digitus volsellaris. Of these digitus is narrower and longer, cuspis is wide and regularly rounded laterally.

The species is allied to *N. flavofasciata* Teranishi from Korea, but fairly well distinguishable by its size, number of segments, antennae and their tiloids, the puncturation of the scape of antenna manubrium and tergit I, as well as the details of coloration, *e. g.* the presence of large yellow spots on the mesonotum.

The remaining three species of this genus are known: from Formosa—*N. taihorina* Bischoff, from Bolivia—*N. schulzi* Bischoff, and from Columbia (Bogota)—*N. fusciatipennis* Shulz. The area of the genus in question including the tropical part of South America and East Asia is thus a very characteristic one. A similar distribution is known for the genus *Lycogaster* allied to *Nanogonalos* (USA, Mexico, Cuba, Burma, Celebes, Java). The same is true of the genus *Orthogonalos* (USA, Bolivia, Formosa, Madagascar), genus *Poocilogonalos* (Korea, Japan, Formosa, Burma, Malacca, Accam, Sumatra, Java, India, Ceylon). The distribution of these genera, as well as the presence in the subfamily *Trigonaloidae* of tropical genera (*Trigonalos*, *Discenea*, *Stygnogonalos*, *Ischnogonalos*, *Lycogastroides*, *Lycogonalos*, *Tapinogonalos* and others) points to the tropical character of the whole subfamily, for which *Pseudogonalos* genus with *P. hahni* widely spread in Palaearctic is the only exception to the general rule.

3. *Pseudogonalos hahni* (Spinola). The species is known from England, Belgium, France, Germany, Switzerland, Hungary, Poland, Japan and also from a number of localities of the USSR: Kharkov (Jaroshevsky, 1889), Ala-Tau (Schulz, 1910), Ural Mountains (Eversmann, 1849), Altai Mountains (Enderlein, 1905). The points to be added to those just mentioned are: Belkino of Yaroslav district, 1♀, 1♂(N. Kokuev); Vladimir, 2♀, 1♂(N. Kasansky); Gremyachka and Kasachii of Rannenburg district, 4♀, 1♂ (A. Semenov-Tyan-Schansky), Suchan of Primorskaya district, 1 (R. Malaise).

The second species of this genus—*P. harmandi* Schulz—is known from Darjeling. One may therefore suppose that like the genus *Lycogaster* the migration of this species towards the North is due to the distribution of the hosts belonging to the genera *Ophion* and *Trogus* (Bischoff) ([3]). The suggestion that the Palaearctic *Trigonaloidae* are relicts of the Tertiary period seems to be quite probable. The question as to whether *Ichneumonidae* or *Vespidae* were the initial hosts of the family, cannot be answered yet. The parasitism of the three subfamilies on *Vespidae* and the tropical character of their distribution show that the ancient hosts of the family were *Vespidae* rather then *Ichneumonidae*. On the other hand, the taxonomic relation of the family, as well as its unquestionable relation to the lower *Hymenoptera* seem rather to indicate that the relations here are the opposite ones. The fact that it is parasitic of *Ichneumonidae* (hyperparasitism) seems to point not only to an extraordinary complexity of the evolution, but to a secondary nature of the parasitic habit also.

As shown by morphological analysis, the family under consideration is poor in external ectosomatic characters. This is peculiar of a number of parasitic groups of *Hymenoptera* in general. However, genera and subfamilies sex characters in *Trigonaloidae* are extremely poor in morphological traits and are very limited, indeed. The differences in the structure of the sex apparatus of the males are also insignificant, as shown by a comparison of the copulatory apparatus and adjacent sternites in *Nanogonalos mongolicus* and *Pseudogonalos hahni* which are the representatives of two subfamilies. The phylogenetic value of the characters of the sex system is unquestionable. The investigation of the copulative apparatus is still the most conclusive demonstration of the formerly disputable problem as to whether the family *Trigonaloidae* belongs to lower *Hymenoptera* (*Terebrantes*). The general appearance of the copulatory apparatus, the structure of stipites and volsellae, *etc.* are typical enough.

On the basis of his comparative analysis of the wing veination and the structure of the copulatory apparatus of the males, Telenga ([4]) suggests to differentiate three phyletic branches in the suborder *Terebrantes* (*Heterophaga*) and to unite *Trigonaloidae* and *Ichneumonidae* into one most ancient and chief branch of the suborder. A decisive argument in the structure of the copulatory apparatus is the presence of a fairly developed volsella with two independent parts—distiovorsella and basivorsella Peck ([5]) and Telenga or Snodgrass ([6]) lamina, cuspis volsellaris and digitus volsellaris. Peck recognized the systematic value of IX sternit and postgenital tergits in *Ichneumonidae*. The form of IX sternit is, on Pratt's opinion ([7]), the best demonstration of primitivity of tribe *Ichneumonini*. The close genetic relation between *Trigonaloidae* and *Ichneumonidae* is unquestionable from this point of view.

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REFERENCES

[1] H. Bischoff, Trigonaloidae in Catalogus Hymenopterorum, 1938. [2] Brues, J. New York Ent. Soc., XVIII (1927). [3] H. Bischoff, Biologie der Hymenopteren, 1927. [4] Telenga, C. R. Acad. Sci. URSS, XXX, No. 9 (1941). [5] Peck, Canad. J. Research., Sec. D, 15 (1937). [6] Snodgrass, Smithson Miscell. Coll., **99**, No. 14 (1941). [7] Pratt, Ann. Ent. Soc. Amer., 32 (1939).

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HELMINTHOLOGY

ON THE TAXONOMIC POSITION OF *HAEMATOTREPHUS FODIENS* LINTON, 1928 WITHIN THE SYSTEM OF TREMATODES

By K. I. SKRJABIN, Member of the Academy

Linton [1] described in 1928 by the name of *Haematotrephus fodiens* a trematode parasitic of *Gavia immer*, which was found by him in two morphological modifications, one of which was freely localized in the intestine of these birds, while the other was contained in cysts in their pancreases.

An analysis of Linton's work has led me to the following conclusions:

1. The two forms described by Linton by the name *H. fodiens* as modifications of a single species were, as a matter of fact, two quite independent species of trematodes, representatives of two different families.

2. The form found by Linton freely localized in the intestine of the loon is an unquestionable representative of the family *Cyclocoeliidae*: it must keep the name *Haematotrephus fodiens* Linton, 1928.

3. The form discovered by Linton in the cysts of the pancreases of those birds is a typical representative of an absolutely different family, that of *Opisthorchidae*.

4. After having compared the trematode described by Linton from the pancreas of *Gavis immer* with other representatives of the family *Opisthorchidae*, with the aim to settle its taxonomic position within the system of trematodes, I am brought to regard it as a representative of an independent new genus of the subfamily *Opisthorchinae*, for which I suggest the name of *Erschoviorchis* nov. gen.

5. In so far as this trematode cannot be allowed to keep its specific name «*fodiens*» it was given by Linton, I suggest for it the name *Erschoviorchis lintoni* nom. nov.

6. I suggest also the following diagnosis of the genus *Erschoviorchis*.

Diagnosis. *Opisthorchinae*, body elongated, gradually expanding towards the caudal end. Posterior end of the body 16 times broader than the anterior one (0.80 mm—0.05 mm, respectively). Oral sucker reduced. Oral aperture leads into a cylindrical canal—the pre-pharynx, passing into the pharynx. Oesophage present. Intestinal trunks extend all along the body, gradually expanding towards the posterior end of the body, where they bend inwards, their blind ends coming into contact at the median line.

There is a small abdominal sucker, with the sexual apertures located anteriorly to it. The sexual glands are concentrated at the posterior end of the body. The lobal seminal glands lie slantwise with respect to each other, the anterior

being contiguous to the medial edge of the left intestinal trunk, the posterior, to the edge of the right intestinal trunk. The ovary, of an irregular oval shape, is localized medially, anteriorly to the testicles, in the interspace between the intestinal trunks. The yolk-sacs are poorly developed, situated in the posterior part of the body. Uterus powerful in development, occupying by its ascending single branch the space between the posterior end of the ovary and the sexual aperture.

Its coils have a regular transverse direction; the cross-section of the uterus becomes gradually smaller towards its distal end. Along with the loops of the uterus, partly interlaced with it, extends to the sexual aperture a long, twisted *vas. deferens*. Eggs small, numerous, 0.024×0.013 mm in size. Parasitic of the pancreas of *Colymbidae*.

The type-species, and the single as yet, *Erschoviorchis lintoni* Skrjabin, 1945 (syn. *Haematotrephus fodiens* Linton, 1928, ex parte).

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REFERENCES

[1] E. Linton, Proc. U. S. Nation. Mus., Wash., 73, Art. 1, pp. 2—4 (1928).